Name-Surname : **Email :**
No : **Signature :**

Midterm 2 (12.12.2018) (09:30-11:00)

BLM3590 – Statistical Data Analysis

Q1(20)	Q2(20)	Q3(20)	Q4(15)	Q5(10)	Q6(15)	Q7(00)	Q8(00)	Q9(00)	Q10(00)	Total(100)

Q01. Determine whether the following statements are correct or not by placing **F**(alse) or **T**(rue) in parenthesis. (20)

a. The probability distribution of a discrete random variable is fully defined by the **probability mass function**. (T)

b. A **hypothesis** is a testable statement about the relationship between two or more variables. (T)

c. The theoretical (population) mean of a random variable ***Y*** with **Binomial(*n*,*θ*)** distribution is $\boldsymbol{\mu = \sqrt{n}\theta}$. (F)

d. If $\boldsymbol{p_{obs}}$ is less than the assumed cutoff, the data provides statistically significant evidence against $\boldsymbol{H_0}$. (T)

e. In normal distribution **85%** of values fall within **2** standard deviation of the mean. (F)

===

Q02. Consider the problem of estimating the proportion of people who regularly smoke. We use X to denote smoking status and μ to denote the population proportion of people who smoke.

a. We hypothesize that the population proportion is less than 0.2. Write down the null and alternative hypotheses. (05)

b. Suppose that we interviewed 150 people and found that 27 of them smoke regularly. Find the z-score for the test statistic. (08)

c. Evaluate the null hypothesis (find the p-value) and decide whether we can reject the null hypothesis at 0.1 confidence level or not. (Use z-table to estimate z) (07)

----

a. The null hypothesis is H_0 : $\mu_0 = 0.2$, the alternative hypothesis is H_A : $\mu_0 < 0.2$

b. $n = 150$, $p = \frac{27}{150} = 0.18$, $z = \frac{p-\mu_0}{\sqrt{\mu_0(1-\mu_0)/n}} = \frac{0.18-0.2}{\sqrt{0.2(1-0.2)/150}} = \frac{-0.02}{\sqrt{0.16/150}} = -0.61$

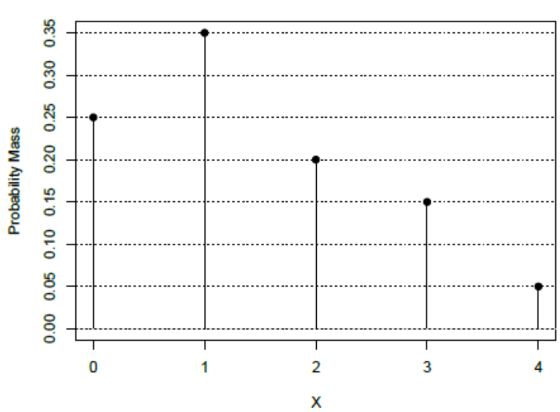
c. Using the standard normal distribution, $p_{obs} = P(Z \leq -0.61|H_0) = 0.27$

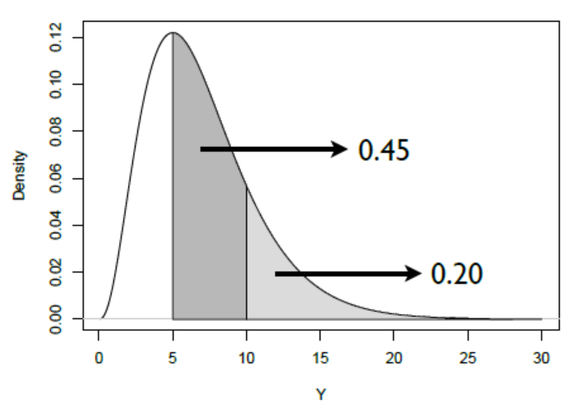
Therefore, we fail to reject H_0 at significance level 0.1.

===

Q03. Considering the following two plots (left panel is the probability mass distribution, right panel is the probability density distribution), determine the following probabilities: (15)

a. $P(X < 3)$ b. $P(1 < X \leq 4)$ c. $P(Y > 5)$ d. $P(Y < 10)$ e. $P(5 < Y) \cap P(Y < 10)$





----

a. $P(X < 3) = 0.80$ b. $P(1 < X \le 4) = 0.40$ c. $P(Y > 5) = 0.45 + 0.20 = 0.65$

d. $P(Y < 10) = 1 - 0.20 = 0.80$ e. $P(5 < Y) \cap P(Y < 10) = 0.45$

===

Q4. A simple random sample of 64 men has a sample mean foot length of 27.5 cm. Assuming that the standard deviation of foot lengths for all men is 2 cm,

a. write the 95% confidence interval for the mean foot length of all men. (10)

$\bar{x} = 27.5$ cm, $\sigma = 2$ cm z_{crit} for 95% CI → $z_{\text{crit}} = 1.96$

95% CI for the population mean is

$$\left[\bar{x} - z_{crit} \times \sigma/\sqrt{n}, \bar{x} + z_{crit} \times \sigma/\sqrt{n}\right] = \left[\bar{x} - 1.96 \times 2/\sqrt{64}, \bar{x} + 1.96 \times 2/\sqrt{64}\right] = \left[\bar{x} - 1.96 \times 2/8, \bar{x} + 1.96 \times 2/8\right] =$$

$$\left[\bar{x} - 1.96 \times 0.25, \bar{x} + 1.96 \times 0.25\right] = \left[\bar{x} - 0.49, \bar{x} + 0.49\right] = \left[27.5 - 0.49, 27.5 + 0.49\right] = \left[27.01, 27.99\right]$$

b. What is the upper limit of this interval, in centimetres? (05)

27.99 cm

===

Q5. The random variable X has a Normal distribution with mean 60 and standard deviation 10. One of the following probabilities is also equal to $P(40 < X \le 48)$. Which one? Explain (10)

a. $P(72 < X \le 80)$ b. $P(64 < X \le 72)$ c. $P(50 < X \le 58)$ d. $P(80 < X \le 88)$ e. $P(56 < X \le 64)$

The normal density curve is symmetric about the mean (60) and (72,80) is the interval symmetrically opposite to (40, 48). Correct answer is **a**.

===

Q6. The random variable *Y* has the distribution shown below:

Value	**1**	**4**
Probability	**0.2**	**0.8**

a. What is the mean of *Y*? (06)

The mean of *Y* is

$1 \times 0.2 + 4 \times 0.8 = 3.4$.

b. What is the variance of *Y* ? (09)

The variance of *Y* is

$0.2 \times (1 - 3.4)^2 + 0.8 \times (4 - 3.4)^2 = 1.44$