

1.1 Determine whether or not the signals below are periodic and, for each signal that is periodic, determine the fundamental period.

- (a) $x(n) = \cos(0.125\pi n)$
 (b) $x(n) = \operatorname{Re}\{e^{jn\pi/12}\} + \operatorname{Im}\{e^{jn\pi/18}\}$
 (c) $x(n) = \sin(\pi + 0.2n)$
 (d) $x(n) = e^{j\frac{\pi}{16}n} \cos(n\pi/17)$
 (a) Because $0.125\pi = \pi/8$, and

$$\cos\left(\frac{\pi}{8}n\right) = \cos\left(\frac{\pi}{8}(n+16)\right)$$

$x(n)$ is periodic with period $N = 16$.

(b) Here we have the sum of two periodic signals,

$$x(n) = \cos(n\pi/12) + \sin(n\pi/18)$$

with the period of the first signal being equal to $N_1 = 24$, and the period of the second, $N_2 = 36$. Therefore, the period of the sum is

$$N = \frac{N_1 N_2}{\operatorname{gcd}(N_1, N_2)} = \frac{(24)(36)}{\operatorname{gcd}(24, 36)} = \frac{(24)(36)}{12} = 72$$

(c) In order for this sequence to be periodic, we must be able to find a value for N such that

$$\sin(\pi + 0.2n) = \sin(\pi + 0.2(n + N))$$

The sine function is periodic with a period of 2π . Therefore, $0.2N$ must be an integer multiple of 2π . However, because π is an irrational number, no integer value of N exists that will make the equality true. Thus, this sequence is aperiodic.

(d) Here we have the product of two periodic sequences with periods $N_1 = 32$ and $N_2 = 34$. Therefore, the fundamental period is

$$N = \frac{(32)(34)}{\operatorname{gcd}(32, 34)} = \frac{(32)(34)}{2} = 544$$

3.1 Consider the discrete-time sequence

$$x(n) = \cos\left(\frac{n\pi}{8}\right)$$

Find two different continuous-time signals that would produce this sequence when sampled at a frequency of $f_s = 10$ Hz.

A continuous-time sinusoid

$$x_a(t) = \cos(\Omega_0 t) = \cos(2\pi f_0 t)$$

that is sampled with a sampling frequency of f_s results in the discrete-time sequence

$$x(n) = x_a(nT_s) = \cos\left(2\pi \frac{f_0}{f_s} n\right)$$

However, note that for any integer k ,

$$\cos\left(2\pi \frac{f_0}{f_s} n\right) = \cos\left(2\pi \frac{f_0 + kf_s}{f_s} n\right)$$

Therefore, any sinusoid with a frequency

$$f = f_0 + kf_s$$

will produce the same sequence when sampled with a sampling frequency f_s . With $x(n) = \cos(n\pi/8)$, we want

$$2\pi \frac{f_0}{f_s} = \frac{\pi}{8}$$

or

$$f_0 = \frac{1}{16} f_s = 625 \text{ Hz}$$

Therefore, two signals that produce the given sequence are

$$x_1(t) = \cos(1250\pi t)$$

and

$$x_2(t) = \cos(21250\pi t)$$

Supplementary Problems

A/D and D/A Conversion

- 3.27 Find two different continuous-time signals that will produce the sequence

$$x(n) = \cos(0.15n\pi)$$

when sampled with a sampling frequency of 8 kHz.

- 3.28 If the Nyquist rate for $x_a(t)$ is Ω_s , find the Nyquist rate for (a) $x^2(2t)$, (b) $x(t/3)$, (c) $x(t) * x(t)$.
- 3.29 A continuous-time signal $x_a(t)$ is known to be uniquely recoverable from its samples $x_a(nT_s)$ when $T_s = 1$ ms. What is the highest frequency in $X_a(f)$?
- 3.30 Suppose that $x_a(t)$ is bandlimited to 8 kHz (that is, $X_a(f) = 0$ for $|f| > 8000$). (a) What is the Nyquist rate for $x_a(t)$? (b) What is the Nyquist rate for $x_a(t) \cos(2\pi \cdot 1000t)$?
- 3.31 Let $x_a(t) = \cos(650\pi t) + 2 \sin(720\pi t)$. (a) What is the Nyquist rate for $x_a(t)$? (b) If $x_a(t)$ is sampled at twice the Nyquist rate, what are the frequencies of the sinusoids in the sampled sequence?
- 3.32 If a continuous-time filter with an impulse response $h_a(t)$ is sampled with a sampling frequency of f_s , what happens to the cutoff frequency ω_c of the discrete-time filter as f_s is increased?
- 3.33 A complex bandpass signal $x_a(t)$ with $X_a(f)$ nonzero for $10 \text{ kHz} < f < 12 \text{ kHz}$ is sampled at a sampling rate of 2 kHz. The resulting sequence is

$$x(n) = \delta(n)$$

What is $x_a(t)$?

- 3.34 If the highest frequency in $x_a(t)$ is $f = 8$ kHz, find the minimum sampling frequency for the bandpass signal $y_a(t) = x_a(t) \cos(\Omega_0 t)$ if (a) $\Omega_0 = 2\pi \cdot 20 \cdot 10^3$ and (b) $\Omega_0 = 2\pi \cdot 24 \cdot 10^3$.
- 3.35 The continuous-time signal $x_a(t) = 7.25 \cos(2000\pi t)$ is sampled at a sampling frequency of 8 kHz and quantized with a resolution $\Delta = 0.02$. How many bits are required in the A/D converter to avoid clipping $x_a(t)$?

Answers to Supplementary Problems

- 3.27 $x_1(t) = \cos(1200\pi t)$ and $x_2(t) = \cos(17200\pi t)$.
- 3.28 (a) $4\Omega_s$. (b) $\Omega_s/3$. (c) Ω_s .
- 3.29 500 Hz.
- 3.30 (a) 16 kHz. (b) 18 kHz.
- 3.31 (a) 720 kHz. (b) $\omega_1 = 65\pi/142$ and $\omega_2 = \pi/2$.
- 3.32 ω_c decreases.
- 3.33 $x_a(t) = \frac{1}{2000} \frac{\sin(2000\pi t)}{\pi t} e^{j2\pi(11000)t}$.
- 3.34 (a) 56 kHz. (b) 32 kHz.
- 3.35 10 bits.