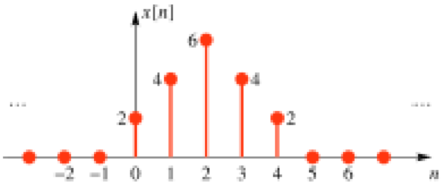
Q13

Determine the output of a *centralized averager*

$y[n]=(1/3)\,(x[n+1]+x[n]+x[n-1])$

for the following input. Is this filter causal or noncausal?

What is the support of the output for this input?



A13

$y[0] = \frac{1}{3}(x[1] + x[0] + x[-1]) = \frac{1}{3}(4+2+0) = 2$

$y[-1] = \frac{1}{3}(x[0] + x[1] + x[2]) = \frac{1}{3}(2+0+0) = 2/3$

$y[-2] = 0$

$y[1] = \frac{1}{3}(x[2] + x[1] + x[0]) = \frac{1}{3}(6+4+2) = 4$

Make a table:

n	≤-2	-1	0	1	2	3	4	5	6	≥7
x[n]	0	0	2	4	6	4	2	0	0	0
y[n]	0	2/3	2	4	14/3	4	2	2/3	0	0

Since y[n] starts before x[n] ⇒ NOT causal

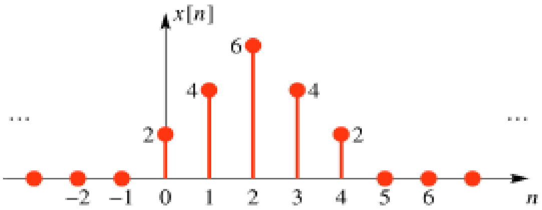
SUPPORT is:

$-1 \le n \le 5$

Q14

Compute the output $y[n]$ for the length-4 filter whose coefficients are $\{b_k\}=\{3, -1, 2, 1\}$. Use the the following signal as input. Verify that the answers tabulated here are correct, then fill in the missing values.

n	$n<0$	0	1	2	3	4	5	6	7	8	$n>8$
$x[n]$	0	2	4	6	4	2	0	0	0	0	0
$y[n]$	0	6	10	18	?	?	?	8	2	0	0



A14

$y[n] = \underset{b_0}{3}x[n] \underset{b_1}{-} x[n-1] + \underset{b_2}{2}x[n-2] \underset{b_3}{+} x[n-3]$

$y[2] = 3x[2] - x[1] + 2x[0] + x[-1]$
$= 3(6) - 4 + 2(2) + 0 = 18$ ✓

$y[3] = 3x[3] - x[2] + 2x[1] + x[0]$
$= 3(4) - 6 + 2(4) + 2 = \boxed{16}$

$y[4] = 3(2) - 4 + 2(6) + 4 = \boxed{18}$

$y[5] = 3(0) - 2 + 2(4) + 6 = \boxed{12}$

$y[6] = 3(0) - 0 + 2(2) + 4 = 8$ ✓

Q15

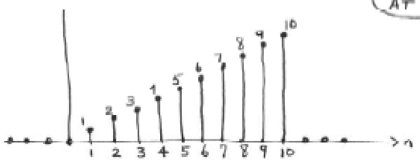
Determine and plot the impulse response of the FIR system

$$y[n] = \sum_{k=0}^{10} k x[n-k]$$

A15

$$y[n] = \sum_{k=0}^{10} k\,\delta[n-k] = 0\cdot\delta[n] + \delta[n-1] + 2\,\delta[n-2] + \ldots + 10\,\delta[n-10]$$

IMPULSE AT n=2



b. The same ADC is used to record a speech signal for 2 minutes. If the sampling frequency of the ADC is 20 kHz (k samples/second), calculate the required memory space (interms of the byte) to store the speech signal. (05)

required memory space $= 2 \times 60 \times 20000 = 2400000$ bytes

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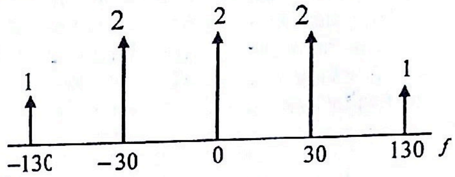
Q4. Frequency spectrum of a signal is given as the following:



a. Write an equation for the signal ($x(t)$) defined by this frequency spectrum. (05)

$$x(t) = 2 + 4\cos(2\pi 30t) + 2\cos(2\pi 130t)$$
$$= 2 + 4\cos(60\pi t) + 2\cos(260\pi t)$$

b. Write $x[n]$ after the signal digitized by an ADC with a sampling frequency of 100 Hz. (05)

$$x[n] = x[nT_s] = 2 + 4\cos(60\pi n T_s) + 2\cos(260\pi n T_s) \quad , \quad T_s = \frac{1}{f_s} = \frac{1}{100}$$
$$x[n] = 2 + 4\cos\left(\frac{60\pi n}{100}\right) + 2\cos\left(\frac{260\pi n}{100}\right)$$
$$x[n] = 2 + 4\cos(0{,}6\pi n) + 2\cos(2{,}6\pi n) =$$
$$x[n] = 2 + 4\cos(0{,}3 \times 2\pi n) + 2\cos(1{,}3 \times 2\pi n)$$

c. Is there any aliasing in (b)? If there is prove it. (05)

$$x[n] = 2 + 4\cos(0{,}6\pi n) + 2\cos((2 + 0.6)\pi n)$$
$$= 2 + 4\cos(0{,}6\pi n) + 2\cos(2\pi n + 0.6\pi n)$$
$$= 2 + 4\cos(0{,}6\pi n) + 2\underbrace{\cos(2\pi n)}_{1}\cos(0{,}6\pi n) - 2\underbrace{\sin(2\pi n)}_{0}\sin(0{,}6\pi n)$$
$$= 2 + 4\cos(0{,}6\pi n) + 2\cos(0{,}6\pi n) = \underline{\underline{2 + 6\cos(0{,}6\pi n)}}$$

So, there is an aliasing

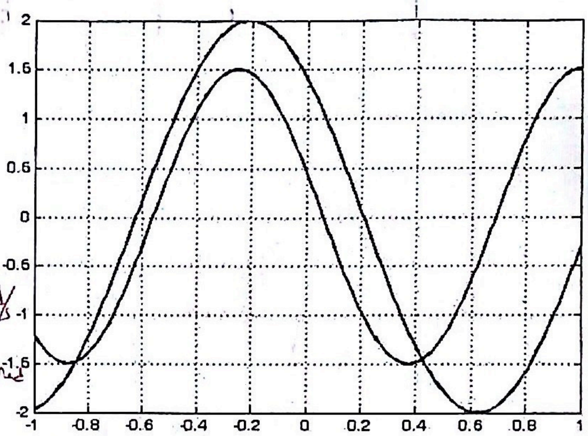
d. What happens to the signal ($x(t)$) when the sampled signal in (b) is reconstructed? (05)

$$x(t) = x\left[\frac{t}{T_s}\right] = 2 + 6\cos\left(\frac{0.6\pi t}{0.01}\right) = 2 + 6\cos(60\pi t)$$

$$x(t) = 2 + \cos(2\pi\, 30 t)$$

Because of aliasing, signal with 130 Hz is Lost. It appears as a 30 Hz signal. Therefor, magnitude of 30 Hz signal became 6.

Q5. Find the three important parameters amplitude ***A***, phase ***φ*** and fundamental angular frequency ***ω₀*** which define a particular sinusoid for the two signals on the following graph. Take time delays approximately. (15)



Signal 1:

$A = 1.5$, $T \cong 1.25$ s, $w_0 = \frac{2\pi}{T} = 1.6\pi$ rad/s

$\varphi \cong 0.4\pi = -w_0 t_m = -1.6\pi(-0.25) = 0.4\pi$ rad

$t_m = -0.25$ s

Signal 2:

$A = 2$, $T \cong 1.6$ s , $w_0 = \frac{2\pi}{T} = 1.25\pi$ rad/s , $\varphi = 0.25\pi$ rad

$t_m = -0.2$ s $\quad \varphi = -w_0 t_m = -1.25\pi \times (-0.2) \cong 0.25\pi$ rad $= \frac{\pi}{4}$ rad.

Q6. A Periodic signal is defined by the equation

$$x(t) = 2 + 4\cos\left(40\pi t - \frac{1}{5}\pi\right) + 3\sin(60\pi t) + 4\cos\left(120\pi t - \frac{1}{3}\pi\right)$$

a. Determine the fundamental frequency ***ω₀***, the fundamental period ***T₀***, and coefficients ***aₖ*** in the Fourier representation for that signal. (10)

$$x(t) = 2 + \frac{4}{2}\left(e^{j(40\pi t - \frac{\pi}{5})} + e^{-j(40\pi t - \frac{\pi}{5})}\right) + \frac{3}{2j}\left(e^{j60\pi t} - e^{-j60\pi t}\right) + \frac{4}{2}\left(e^{j(120\pi t - \frac{\pi}{3})} + e^{-j(120\pi t - \frac{\pi}{3})}\right)$$

$$x(t) = 2 + 2e^{-j\frac{\pi}{5}}e^{j40\pi t} + 2e^{j\frac{\pi}{5}}e^{-j40\pi t} + 1.5e^{-j\frac{\pi}{2}}e^{j60\pi t} + 1.5e^{j\frac{\pi}{2}}e^{-j60\pi t} + 2e^{-j\frac{\pi}{3}}e^{j120\pi t} + 2e^{j\frac{\pi}{3}}e^{-j120\pi t}$$

not: $\frac{1}{j} = -j = e^{-j\frac{\pi}{2}}$, $-e^{-j\frac{\pi}{2}} = j = e^{j\frac{\pi}{2}}$

$f_1 = 20$ Hz, $f_2 = 30$ Hz, $f_3 = 60$ Hz, so $f_0 = 10$ Hz $\Rightarrow T_0 = \frac{1}{10} = 0.1$ s

$w_0 = 20\pi$ rad/s

1. (35 pts.) (a) Compute the following convolution:

$$\{a, b, c, d, e\} * \{e, d, c, b, a\}$$

where a, b, c, d, e are some numbers. Both sequences above start with the n=0 index in time. (15 pts.)

(b) The impulse response of a LTI system is given as $\mathbf{h} = \{1, 2, 1\}$. The output sequence is known to be $\mathbf{y} = \{1, 4, 8, 12, \ldots\}$. Both sequences start with the n=0 index in time. Find the input sequence $x(n)$. (20 pts.)

$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n)$$

Answer:

1. (a) $x = \{\underset{n=0}{a}, b, c, d, e\}$, $y = \{\underset{n=0}{e}, d, c, b, a\}$

$$y(-k) = \{a, b, c, d, \underset{k=0}{e}\}$$

$$z(n) = \sum_{k=-\infty}^{\infty} x(k)\, y(n-k) = \sum_k x(k)\, y(-(k-n))$$

$z(0) = ae$, $z(1) = ad + be$, $z(2) = ac + bd + ce$

$z(3) = ab + bc + cd + de$, $z(4) = a^2 + b^2 + c^2 + d^2 + e^2$,

$z(5) = ba + cb + dc + ed$, $z(6) = ca + db + ec$,

$z(7) = da + eb$, $z(8) = ea$, $z(n) = 0$ for $n < 0$ and $n > 8$.

(b) The freq. response: $H(e^{j\omega}) = \sum_n h(n)\, e^{-j\omega n} = 1 + 2e^{-j\omega} + e^{-j2\omega}$

Output Fourier transform: $Y(e^{j\omega}) = 1 + 4e^{-j\omega} + 8e^{-j2\omega} + 12e^{-j3\omega} + \ldots$

$$= H(e^{j\omega})\, X(e^{j\omega})$$

The input Fourier transform:

$$X(e^{j\omega}) = \text{DTFT}\{x(n)\} = \frac{Y(e^{j\omega})}{H(e^{j\omega})}$$

$1 + 4e^{-j\omega} + 8e^{-j2\omega} + 12e^{-j3\omega} + \ldots$ | $1 + 2e^{-j\omega} + e^{-j2\omega}$

$-\;1 + 2e^{-j\omega} + e^{-j2\omega}$ | $1 + 2e^{-j\omega} + 3e^{-j2\omega} + 4e^{-j3\omega} + \ldots$

$2e^{-j\omega} + 7e^{-j2\omega} + 12e^{-j3\omega} + \ldots$

$-\;2e^{-j\omega} + 4e^{-j2\omega} + 2e^{-j3\omega}$

$3e^{-j2\omega} + 10e^{-j3\omega} + 16e^{-j4\omega} + \ldots$

(2)

$$X(e^{j\omega}) = 1 + 2e^{-j\omega} + 3e^{-j2\omega} + 4e^{-j3\omega} + \cdots$$

$$= \text{DTFT}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

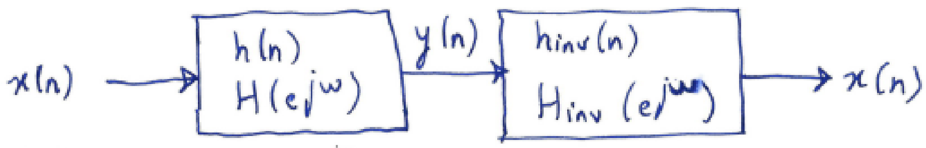
$$\therefore \quad x(n) = \{\underset{\underset{n=0}{\uparrow}}{1}, 2, 3, 4, 5, \cdots\} = \begin{cases} n+1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

2. (35 pts.) Consider a LTI system described by the following difference equation:

y(n) – (3/4) y(n – 1) + (1/8) y(n –2) = 2 x(n – 1)

with y(–1) = y(–2) = 0. x(n) and y(n) denote input and output sequences, respectively.

(a) Find the frequency response $H(e^{jw})$ of this system. (10 pts.)
(b) Find the impulse response h(n) of this system. (Hint: Apply partial fraction expansion before taking the inverse DTFT.) Is this system causal? Is it stable? (10 pts.)
(c) Consider the inverse system of this system:



Find the frequency response $H_{inv}(e^{jw})$ of the inverse system. (7.5 pts.)
(d) Find the impulse response $h_{inv}(n)$ of the inverse system. Is the inverse system causal? Is it stable? (7.5 pts.)

Answer:

2. (a) $y(n) - \frac{3}{4} y(n-1) + \frac{1}{8} y(n-2) = 2\, x(n-1)$

Let's take Fourier tr. of both sides: $Y(e^{j\omega}) = \text{DTFT}\{y(n)\}$,
$X(e^{j\omega}) = \text{DTFT}\{x(n)\}$

and use $\text{DTFT}\{y(n-k)\} = e^{-j\omega k} \cdot Y(e^{j\omega})$:

$$\left[1 - \frac{3}{4} e^{-j\omega} + \frac{1}{8} e^{-j2\omega}\right] Y(e^{j\omega}) = 2e^{-j\omega} X(e^{j\omega})$$

$$\boxed{H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2e^{-j\omega}}{1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}}}$$

(b) $h(n) = \text{IDTFT}\{H(e^{j\omega})\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega$

where $H(e^{j\omega}) = \frac{2e^{-j\omega}}{1-\frac{3}{4}e^{-j\omega}+\frac{1}{8}e^{-j2\omega}} = \frac{2e^{-j\omega}}{(1-\frac{1}{4}e^{-j\omega})(1-\frac{1}{2}e^{-j\omega})}$

$$\frac{1}{(1-\frac{1}{4}e^{-j\omega})(1-\frac{1}{2}e^{-j\omega})} = \frac{2}{1-\frac{1}{2}e^{-j\omega}} - \frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

(3)

$$H(e^{j\omega}) = \frac{4e^{-j\omega}}{1-\frac{1}{2}e^{-j\omega}} - \frac{2e^{-j\omega}}{1-\frac{1}{4}e^{-j\omega}}$$

$$\boxed{h(n) = 4\cdot(1/2)^{n-1}u(n-1) - 2\cdot(1/4)^{n-1}u(n-1),}$$

with $u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$

$h(n) = 0$ for $n<0 \Rightarrow$ the system is <u>causal.</u>

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=1}^{\infty} \left|4\cdot(1/2)^{n-1} - 2\cdot(1/4)^{n-1}\right|$$

$$= \sum_{n=0}^{\infty}\left[4\cdot(1/2)^n - 2\cdot(1/4)^n\right] = 4\cdot\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n - 2\cdot\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n$$

$$= 4\cdot\frac{1}{1-1/2} - 2\cdot\frac{1}{1-1/4} = 8 - 8/3 = 16/3 < \infty$$

$\therefore$ The system is <u>stable.</u>

(c) $H_{inv}(e^{j\omega}) = \dfrac{X(e^{j\omega})}{Y(e^{j\omega})} = \dfrac{1}{H(e^{j\omega})} = \dfrac{1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-j2\omega}}{2e^{-j\omega}}$

$$= \boxed{\frac{1}{2}e^{j\omega} - \frac{3}{8} + \frac{1}{16}e^{-j\omega}}$$

(d) $H_{inv}(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h_{inv}(n)e^{-j\omega n} \Rightarrow h_{inv}(n) = \frac{1}{2}\delta(n+1) - \frac{3}{8}\delta(n) + \frac{1}{16}\delta(n-1)$

$$\boxed{h_{inv}(n) = \begin{cases} 1/2, & n=-1 \\ -3/8, & n=0 \\ 1/16, & n=1 \\ 0, & \text{otherwise} \end{cases}}$$

$h_{inv}(-1) = 1/2 \neq 0$

$\Downarrow$

the system is noncausal.

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(4)

$\sum_n |h(n)| = \frac{1}{2} + \frac{3}{8} + \frac{1}{16} = \frac{15}{16} < \infty \Rightarrow$ the system is stable.

3. (30 pts.) A system with input signal x(n) and output signal y(n), is described as

$$y(k) = x(-\infty) = \lim_{i \to -\infty} x(i)$$

for all k values.

(a) Is this system linear or not? Justify your answer.

(b) Is this system time-invariant or not? Justify your answer.

(c) Let the input be $x(n) = \delta(n)$, the unit sample sequence (discrete-time impulse signal). Find the output, i.e., the impulse response h(n) of this system.

Can the output signal y(n) be computed by convolving the impulse response h(n) with an input signal x(n), if $x(-\infty) \neq 0$?

Answer:

3. (a) $y(k) = \lim_{i \to -\infty} x(i) = x(-\infty) = T\{x(n)\}$

Let $y_1(n) = T\{x_1(n)\}$ and $y_2(n) = T\{x_2(n)\}$ be two input/output pairs.

$y_1(k) = x_1(-\infty) = \lim_{i \to -\infty} x_1(i)$

for all k.

$y_2(k) = x_2(-\infty) = \lim_{i \to -\infty} x_2(i)$

If $x(n) = a x_1(n) + b x_2(n)$ is a new input, the corresponding output will be: $y(k) = x(-\infty) = \lim_{i\to\pm\infty} x(i) = \lim_{i\to\pm\infty} [a x_1(i) + b x_2(i)]$

$$= a\cdot \lim_{i\to\pm\infty} x_1(i) + b\cdot \lim_{i\to\pm\infty} x_2(i)$$

$$= a x_1(-\infty) + b\, x_2(-\infty) = a y_1(k) + b y_2(k)$$

for any a, b. for all k,

$\therefore$ The system is linear.

(b) $y(k) = \lim_{i\to-\infty} x(i) = x(-\infty)$, for all k, for an input/output pair.

Let $x_1(n) = x(n-m)$ be a new input signal to this system, for a fixed integer m.

The corresponding output: $y_1(k) = \lim_{i\to-\infty} x_1(i) = \lim_{i\to-\infty} x(i-m)$
$= x(-\infty)$, for all k.

$y(n-m) = x(-\infty) = \text{constant}$, for all n.

$\therefore$ $y_1(n) = y(n-m) = x(-\infty)$ for any fixed m.

$\therefore$ The system is also time-invariant.

(c) $h(n) = \delta(-\infty) = \lim_{i\to-\infty} \delta(i) = 0$, for all n.

If $x(-\infty) \neq 0$ for an input signal $x(n)$, the output:
$y(n) = x(-\infty) \neq 0$, for all n.

$$x(n) * h(n) = \sum_k \underbrace{h(k)}_{0} x(n-k) = 0, \text{ for all } n.$$

$\therefore$ $y(n) \neq x(n) * h(n)$, if $x(-\infty) \neq 0$,

although the system is a LTI system.

Some LTI systems are so-called "nonconvolutional" systems, i.e., they can't be modeled by convolution operation, although they are LTI systems.

This system is an example of such a nonconvolutional system.

1. (35 pts.) Consider a LTI system described by

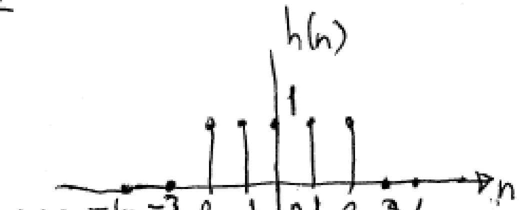
$$y(n) = x(n+2) + x(n+1) + x(n) + x(n-1) + x(n-2),$$

where x(n) and y(n) denote input and output sequences, respectively.

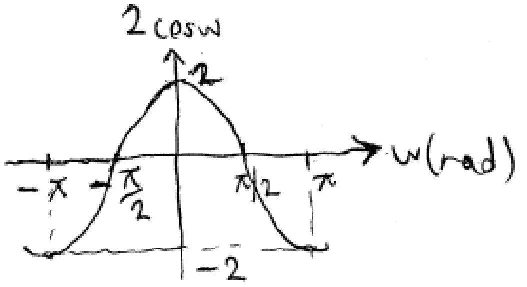
(a) Find and plot impulse response h(n) of this system.
(b) Find the frequency response $H(e^{jw})$ of the system, and plot it for $-\pi \le w \le \pi$ rad.
(c) Is this system stable or not? Is it causal or not?
(d) Find a recursive difference equation expressing this system.
(e) Find a difference equation for the "inverse system" of this system.
(f) Find the impulse response of the inverse system.
(g) Is the inverse system stable? Is it causal?

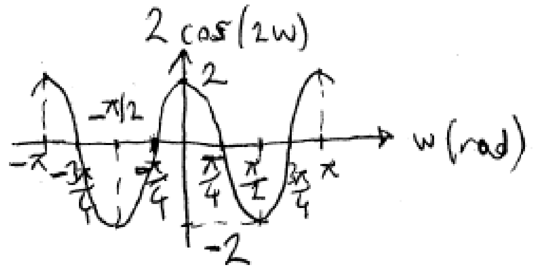
<u>Answer:</u> (a) $h(n) = \delta(n+2) + \delta(n+1) + \delta(n) + \delta(n-1) + \delta(n-2)$

$$= \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$$



(b) $H(e^{jw}) = e^{-j2w} + e^{-jw} + 1 + e^{jw} + e^{j2w} = 1 + 2\cos w + 2\cos(2w),$



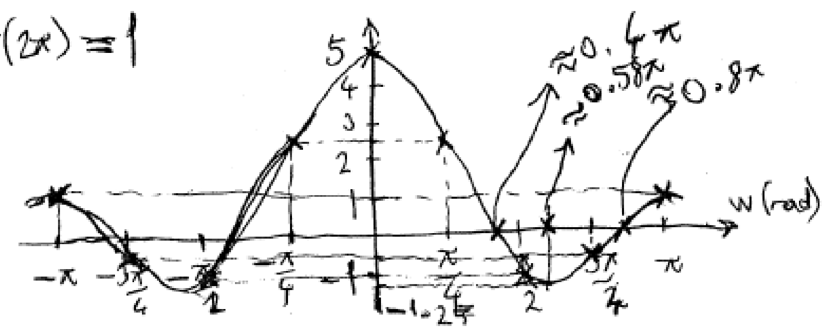


$H(e^{j0}) = 5, \; H(e^{j\pi/4}) = 1 + 2\underbrace{\cos(\pi/4)}_{1/\sqrt{2}} + 2\underbrace{\cos(\pi/2)}_{0} = 1 + \sqrt{2} \approx 2.4142$

$H(e^{j\pi/2}) = 1 + 2\cos(\pi/2) + 2\cos(\pi) = -1$

$H(e^{j3\pi/4}) = 1 + 2\cos(3\pi/4) + 2\cos(3\pi/2) = 1 - \sqrt{2} \approx -0.4142$

$H(e^{j\pi}) = 1 + 2\cos(\pi) + 2\cos(2\pi) = 1$



(c) $\sum_n |h(n)| = 5 < \infty \Rightarrow$ stable system.

$h(-1) \neq 0,\ h(-2) \neq 0 \Rightarrow$ non causal system.

(d) $y(n) = x(n+2) + x(n+1) + x(n) + x(n-1) + x(n-2)$

$y(n+1) = x(n+3) + x(n+2) + x(n+1) + x(n) + x(n-1)$

$\therefore\ y(n+1) - y(n) = x(n+3) - x(n-2) \Rightarrow \boxed{y(n) - y(n-1) = x(n+2) - x(n-3)}$

(e) $y(n) \longrightarrow \boxed{h_{inv}(n)} \longrightarrow x(n)$

$x(n+2) + x(n+1) + x(n) + x(n-1) + x(n-2) = y(n)$

$\boxed{x(n) + x(n-1) + x(n-2) + x(n-3) + x(n-4) = y(n-2)}$

or $x(n+2) - x(n-3) = y(n) - y(n-1) \Rightarrow \boxed{x(n) - x(n-5) = y(n-2) - y(n-3)}$

$x(n)$: output, $y(n)$: input of the inverse system.
Both of them are correct.

(f) $H_{inv}(e^{jw}) = \dfrac{X(e^{jw})}{Y(e^{jw})} = \dfrac{e^{-j2w}}{1 + e^{-jw} + e^{-j2w} + e^{-j3w} + e^{-j4w}}$

$$1 \div (1 + e^{-jw} + e^{-j2w} + e^{-j3w} + e^{-j4w}) = 1 - e^{-jw} + e^{-j5w} - e^{-j6w} + e^{-j10w} - e^{-j11w} + e^{-j15w} - e^{-j16w} + \cdots$$

$$1$$
$$-(1 + e^{-jw} + e^{-j2w} + e^{-j3w} + e^{-j4w})$$
$$-e^{-jw} - e^{-j2w} - e^{-j3w} - e^{-j4w}$$
$$-e^{-jw} - e^{-j2w} - e^{-j3w} - e^{-j4w} - e^{-j5w}$$
$$e^{-j5w}$$
$$e^{-j5w} + e^{-j6w} + e^{-j7w} + e^{-j8w} + e^{-j9w}$$
$$-e^{-j6w} - e^{-j7w} - e^{-j8w} - e^{-j9w}$$
$$-e^{-j6w} - e^{-j7w} - e^{-j8w} - e^{-j9w} - e^{-j10w}$$
$$e^{-j10w}$$
$$\vdots$$

$$\therefore H_{inv}(e^{jw}) = e^{-j2w} - e^{-j3w} + e^{-j7w} - e^{-j8w} + e^{-j12w} - e^{-j13w} + e^{-j17w} - e^{-j18w} + \cdots$$
$$= \sum_{n=-\infty}^{\infty} h_{inv}(n)\, e^{-jwn}$$

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2.(30 pts.) The sequence of daily sunspot (güneş lekesi) numbers is smoothed by taking five-day running totals. For each day we add the sunspot numbers for the preceding and following two days:

$$y(n) = x(n+2) + x(n+1) + x(n) + x(n-1) + x(n-2),$$

where x(n) and y(n) denote the sunspot number observed on day n and the running total value on day n, respectively. Here is a sequence of five-day running totals:

y = {45, 35, 25, 15, 5, $\overset{\downarrow}{0}$, 0, 0, 0, 15, 50, 80, 100, 125, 125, 100, 80, 70, 45, 30, 30, 30, 35, 60, 80, 90, 95, 100, 90, 85, 75}. Find the actual sequence of daily sunspot numbers, **x** .

<u>Hint:</u> Daily sunspot numbers should be nonnegative integers.

<u>Answer:</u> Let the marked index be $n = 0$:

$$y(0) = 0 = x(-2) + x(-1) + x(0) + x(1) + x(2)$$

$$\therefore \quad x(-2) = x(-1) = x(0) = x(1) = x(2) = 0,$$

since $x(n) \geq 0$ and integer always ($x(n)$: # of sunspots on nth day) for all n.

For $n > 2$: $x(n) = y(n-2) - x(n-1) - x(n-2) - x(n-3) - x(n-4)$

can be used to recover $x(n)$ from the above initial values of $x(n)$, recursively.

For $n < 2$: $x(n-4) = y(n-2) - x(n) - x(n-1) - x(n-2) - x(n-3)$

can be used to recover $x(n)$, recursively.

y			45	35	25	15	5	0	0	0	0	15	50	80	100
x	10	10	10	10	5	0	0	0	0	0	0	0	0	15	35

y	125	125	100	80	70	45	30	30	30	35	60	80
x	30	20	25	15	10	10	10	0	0	10	10	15

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y	90	95	100	90	85	75			
x	25	20	20	15	20	15	15	10	...