BLM2041 Signals and Systems

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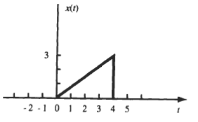
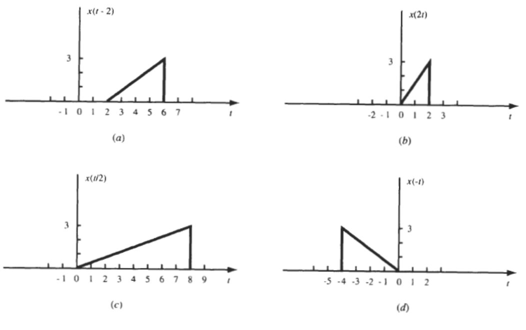
Example 1

- Given the following continuous-time signal $x(t)$;
  - sketch and label each of the following signals.



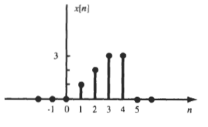
(a) $x(t - 2)$;
(b) $x(2t)$;
(c) $x(t/2)$;
(d) $x(-t)$

Answer 1



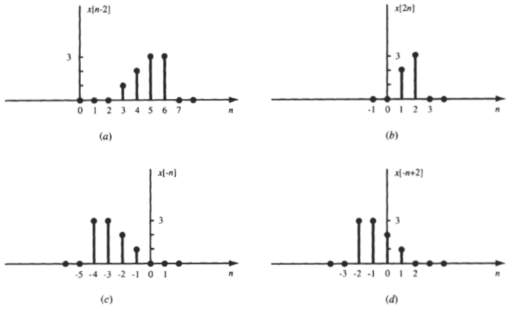
Example 2

- Given the following discrete-time signal $x[n]$;
  - sketch and label each of the following signals.



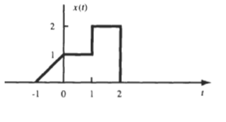
(a) $x[n – 2]$;
(b) $x[2n]$;
(c) $x[-n]$;
(d) $x[-n + 2]$

Answer 2



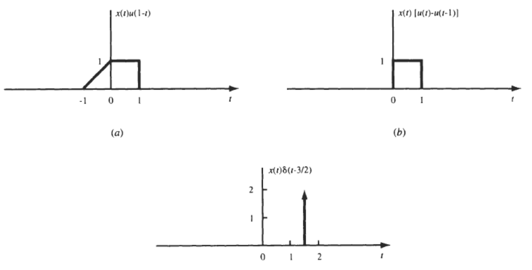
Example 3

- Given the following continuous-time signal $x(t)$;
  - sketch and label each of the following signals.



(a) $x(t)u(1- t)$;
(b) $x(t)\ [u(t) - u(t - 1)]$;
(c) $x(t/2)\delta(t-1.5)$;

Answer 3



Example 4

- Find the energy content of the exponentially decreasing signal $x(t)$

$$x(t) = \begin{cases} e^{-2t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Answer 4

- 1st, compute the square

$$|x(t)|^2 = (e^{-2t})^2 = e^{-4t}$$

- Considering that the signal is zero for t < 0,

$$E = \int_0^\infty |x(t)|^2 dt = \int_0^\infty e^{-4t} dt = -\frac{1}{4}e^{-4t}\Big|_0^\infty$$

$$E = -\frac{1}{4}e^{-4t}\Big|_0^\infty = \left[-\frac{1}{4}e^{-4\times\infty} + \frac{1}{4}e^{-4\times 0}\right]$$

$$E = \left[-\frac{1}{4}\times 0 + \frac{1}{4}\times 1\right] = \frac{1}{4}$$

- The energy is finite,
  - so this is an energy signal.

Example 5

- Let $x(t) = A\cos\omega t$, where A is a positive real constant.
- Find
  - (a) the signal energy over one period
  - (b) the average power of the signal

Answer 5

(a) The period of this signal: $T_0 = \frac{2\pi}{\omega}$

Square of signal: $\cos^2 x = \frac{1 + \cos 2x}{2}$

The energy over one period is

$$E_0 = \int_{-T_0/2}^{T_0/2} |x(t)|^2\, dt = \int_{-T_0/2}^{T_0/2} |A \cos\omega t|^2\, dt = A^2 \int_{-T_0/2}^{T_0/2} \cos^2\omega t\, dt$$

$$E_0 = A^2 \int_{-T_0/2}^{T_0/2} \frac{1 + \cos 2\omega t}{2} dt = \frac{A^2}{2}\int_{-T_0/2}^{T_0/2} dt + \frac{A^2}{2}\int_{-T_0/2}^{T_0/2} \cos 2\omega t\, dt$$

Answer 5

$$\frac{A^2}{2}\int_{-T_0/2}^{T_0/2} dt = \frac{A^2}{2} t\Big|_{-T_0/2}^{T_0/2} = \frac{A^2}{2}\left(\frac{T_0}{2} + \frac{T_0}{2}\right) = \frac{A^2}{2} T_0$$

$$\int_{-T_0/2}^{T_0/2} \cos 2\omega t\, dt = \frac{1}{2\omega} \sin 2\omega t\Big|_{-T_0/2}^{T_0/2}$$

$$= \frac{1}{2\omega}[\sin(\omega T_0) - \sin(-\omega T_0)] = \frac{\sin(\omega T_0)}{\omega}$$

$$\int_{-T_0/2}^{T_0/2} \cos 2\omega t\, dt = \frac{\sin(\omega T_0)}{\omega} = \frac{\sin(2\pi)}{\omega} = 0 \qquad E_0 = \frac{A^2}{2} T_0$$

(b) Average power:

$$P = \frac{E_0}{T_0} = \frac{A^2 T_0/2}{T_0} = \frac{A^2}{2}$$

Example 6

- Consider a signal $x(t) = e^{-|t|}$.
  Determine the energy and power content of this signal.

Answer 6

- Compute the squared modulus of the function

$$|x(t)|^2 = e^{-2|t|} \qquad |x(t)|^2 = \begin{cases} e^{2t} & \text{for } t < 0 \\ e^{-2t} & \text{for } t > 0 \end{cases}$$

- Split the integral into two parts, and perform the calculation

$$E = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{0} e^{2t}\, dt + \int_{0}^{\infty} e^{-2t}\, dt = 2\int_{0}^{\infty} e^{-2t}\, dt = 1$$

- The energy is finite (energy signal)

Answer 6

- To find average power, compute

$$\frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, d = \frac{1}{T}\int_{-T/2}^{T/2} e^{-2|t|}\, dt = \frac{2}{T}\int_{0}^{T/2} e^{-2t}\, dt = \frac{1}{T}\left(1 - e^{-T}\right)$$

- Take the limit:

$$P = \lim_{T\to\infty} \frac{1}{T}\left(1 - e^{-T}\right) = \lim_{T\to\infty} \frac{1}{T} - \lim_{T\to\infty} \frac{e^{-T}}{T}$$

  - The first term vanishes.
  - For the second term, notice that as $T \rightarrow \infty$, $e^{-T} \rightarrow 0$.
    - Therefore the second term vanishes as well, and we have $P = 0$ as expected for an energy signal.

Example 7

- Find the even and odd components of
  $x(t) = 2\cos t - \sin t + 3\sin t \cos t$
- Reminder:

$$x_e(t) = \frac{x(t) + x(-t)}{2} \qquad x_0(t) = \frac{x(t) - x(-t)}{2}$$

Answer 7

- 1st, find $x(-t)$
  $x(-t) = 2\cos(-t) - \sin(-t) + 3\sin(-t)\cos(-t)$
  $x(-t) = 2\cos t + \sin t - 3\sin t \cos t$
- Even component

$$x_e(t) = \frac{x(t) + x(-t)}{2} = \frac{4\cos t}{2} = 2\cos t$$

- Odd component

$$x_0(t) = \frac{x(t) - x(-t)}{2} = \frac{-2\sin t + 6\sin t\cos t}{2} = -\sin t + 3\sin t\cos t$$