Midterm 2 (18.12.2018) (13:00-15:00)

BLM 2401- Signals and Systems

Q1(10)	Q2(10)	Q3(15)	Q4(10)	Q5(10)	Q6(25)	Q7(20)	Q8(00)	Q9(00)	Q10(00)	Total

Q01. Chose (circle) the correct answer from the multiple answers. (10)

i. Convolution property states that the FS of convolved signal is

(a) convolution of FS of two signals
(b) addition of FS of two signals
(c) multiplication of FS of two signals
(d) linear combination of scaled FS of two signals

ii. Sampling a signal is equivalent to multiplying it with

(a) a sync function
(b) a train of impulse
(c) a train of sync functions
(d) a rectangular window

iii. Fourier series representation is used for

(a) Periodic signals
(b) Aperiodic signals
(c) Pseudo-periodic signals
(d) All signals

iv. When impulse is given as input to the system, the output of the system is called

(a) input response
(b) stable response
(c) steady response
(d) impulse response

v. FT of a delta function in time domain is

(a) A constant value 1 for all frequencies
(b) A constant value 2*π for all frequencies
(c) A delta function existing at frequency of zero
(d) A series of delta functions

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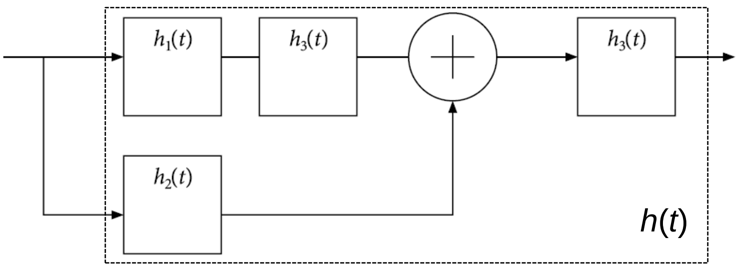
Q02. Consider the configuration shown in the following figure with impulse responses given by $h_1(t)$, $h_2(t)$, $h_3(t)$ for the systems. Find the impulse response of the overall configuration ($h(t)$). (10)



First, find the impulse response of the series system $h_a(t)$ as

$h_a(t) = h_1(t) * h_3(t)$

This series configuration is in parallel with $h_2(t)$. The impulse response of the parallel configuration is

$h_b(t) = [h_a(t) + h_2(t)] = [h_1(t) * h_3(t)] + h_2(t)$

The system with impulse response $h_3(t)$ is connected in series with this parallel configuration. The impulse response of the overall system is then given by

$h(t) = h_b(t) * h_3(t) = \{[h_1(t) * h_3(t)] + h_2(t)\} * h_3(t)$

Answer is:

$\boldsymbol{h(t) = \{[h_1(t) * h_3(t)] + h_2(t)\} * h_3(t)}$

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Q03. Consider a simple second order system with characteristic equation given by $(D^2 + 3D + 2)y(t) = 0$. Find the zero input response if the initial conditions are $y(0) = 0$ and $Dy(0) = 5$. (15)

$(D^2 + 5D + 6) = 0 \quad \Rightarrow \quad (D + 2)(D + 3) = 0$

The roots are $D = -3$ and $D = -2$.

D is of the form $e^{-\lambda_1 t}$ and $e^{-\lambda_2 t}$. The solution can be written as

$y(t) = c_1 e^{-2t} + c_2 e^{-3t}$

This is called as zero input response. We will now apply initial conditions to find the values of c_1 and c_2. Put $t = 0$ in the solution to get

$y(0) = c_1 + c_2 = 0$

Find the derivative of the solution and put $t = 0$ in the equation to get

$D\{y(t)\} = -2c_1 e^{-2t} - 3c_2 e^{-3t} \quad \Rightarrow \quad D\{y(0)\} = -2c_1 - 3c_2 = 5$

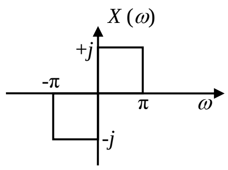
$c_1 + c_2 = 0 \quad \Rightarrow \quad c_1 = -c_2$

$-2c_1 - 3c_2 = 5 \quad \Rightarrow \quad 2c_2 - 3c_2 = 5 \quad \Rightarrow \quad c_2 = -5, \qquad c_1 = -c_2 = 5$

The solution is

$$\boldsymbol{y(t) = (5e^{-2t} - 5e^{-3t})u(t)}$$

Q04. Frequency response ($X(\omega)$) of $x(t)$ is given by the following figure. Determine $x(t)$. (10)



$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\pi}^{0}(-j)e^{j\omega t}d\omega + \frac{1}{2\pi}\int_{0}^{+\pi}(+j)e^{j\omega t}d\omega = -\frac{j}{2\pi jt}e^{j\omega t}\Big|_{-\pi}^{0} + \frac{j}{2\pi jt}e^{j\omega t}\Big|_{0}^{+\pi}$$

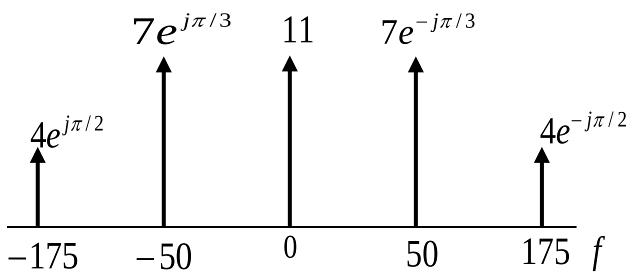
$$x(t) = \frac{1}{2\pi t}\left[-1 + e^{-j\pi t} + e^{j\pi t} - 1\right] = \frac{1}{\pi t}\left[\frac{e^{-j\pi t} + e^{j\pi t} - 2}{2}\right] = \frac{1}{\pi t}\left[\frac{e^{j\pi t} + e^{-j\pi t}}{2} - 1\right]$$

$$x(t) == \frac{1}{\pi t}\left[\cos \pi t - 1\right] = \frac{\cos \pi t}{\pi t} - \frac{1}{\pi t}$$

Q05. A signal $x(t)$ has the two-sided spectrum representation shown in the figure on the right.

a. Write an equation for $x(t)$ as a sum of cosines. (05)

b. Is $x(t)$ a periodic signal? If so determine its fundamental frequency. (05)



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a.

$x(t) = 11 + 14\cos(100\pi - \pi/3) + 8\cos(350\pi - \pi/2)$

b.

Yes it is periodic since the frequencies are the multiple times of a fundamental frequency.

The fundamental frequency f_0 is given by

$$\boldsymbol{f_0 = \gcd(50, 175) = 25\ Hz}$$

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Q06. A LTI filter is described as

$$y[n] = x[n] + 2x[n-1] + x[n-2]$$

a) Obtain an expression for the frequency response of this system, i.e., $H(j\omega)$ =? (04)

b) Sketch the frequency response (magnitude and phase) as a function of frequency. (05)

c) Determine the output when the input is $x[n] = 10 + 4\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)$. (07)

d) Determine the output when the input is the unit impulse response, $\delta[n]$. (04)

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a.

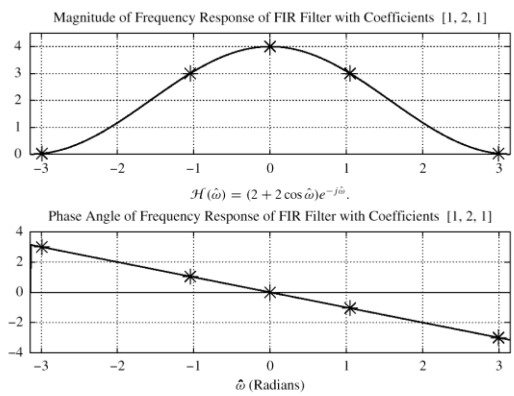
$$\begin{aligned} H(e^{j\hat{\omega}}) &= 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} \\ &= e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}) \\ &= e^{-j\hat{\omega}}(2 + 2\cos\hat{\omega}) \end{aligned}$$

b.

Since $(2 + 2\cos\hat{\omega}) \geq 0$

Magnitude is $\left|H(e^{j\hat{\omega}})\right| = (2 + 2\cos\hat{\omega})$

and Phaseis $\angle H(e^{j\hat{\omega}}) = -\hat{\omega}$



c.

$$x[n] = 10 + 4\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right) = 10 + 2e^{j(\frac{\pi n}{2}+\frac{\pi}{4})} + 2e^{-j(\frac{\pi n}{2}+\frac{\pi}{4})}.$$

$$y[n] = 10H(0) + H(\tfrac{\pi}{2})2e^{j(\frac{\pi n}{2}+\frac{\pi}{4})} + H(\tfrac{-\pi}{2})2e^{-j(\frac{\pi n}{2}+\frac{\pi}{4})}.$$

$H(0) = 4e^{j0}$ $\quad H(\pi/2) = 2e^{-j\pi/2}$ $\quad H(-\pi/2) = 2e^{j\pi/2}$

$$y[n] = 40 + 2e^{-j\frac{\pi}{2}}2e^{j(\frac{\pi n}{2}+\frac{\pi}{4})} + 2e^{j\frac{\pi}{2}}2e^{-j(\frac{\pi n}{2}+\frac{\pi}{4})} = 40 + 4e^{j(\frac{\pi n}{2}-\frac{\pi}{4})} + 4e^{-j(\frac{\pi n}{2}-\frac{\pi}{4})} = 40 + \cos(\frac{\pi n}{2} - \frac{\pi}{4})$$

$$\boldsymbol{y[n] = 40 + \cos(\frac{\pi n}{2} - \frac{\pi}{4})}$$

d.

$x[n] = \delta[n]$ ➔ $\boldsymbol{y[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]}$

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Q07. A periodic function $x(t)$ is given by the following equation .

$$x(t)=\begin{cases}-1 & 0<t<1\\ +1 & 1<t<2\end{cases}$$

a. Plot the function $x(t)$. (04)
b. Determine the fundamental frequency of the signal. (04)
c. Find D.C. component of the signal (a_0). (04)
d. Find the Fourier series components (a_k). (08)
e. Plot the first three frequency spectrum ($a_{\pm1}$, $a_{\pm2}$, $a_{\pm3}$). (05)

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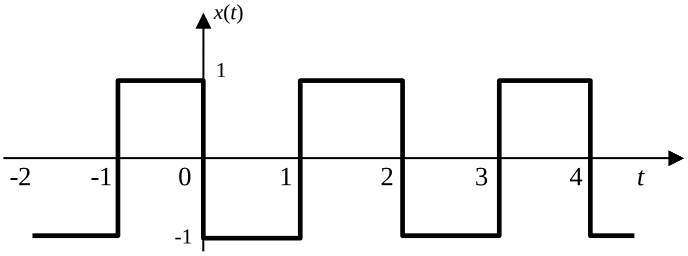
a.



b. $T_0 = 2$ s; $f_0 = 1/T_0 = 1/2 = 0.5$ Hz

$f_0 = 0.5$ Hz

c. $$a_0=\frac{1}{T_0}\int_0^{T_0}x(t)dt=\frac{1}{2}\left\{\int_0^1(-1)dt+\int_1^2 dt\right\}=\frac{1}{2}\times\left\{-t\Big|_0^1+t\Big|_1^2\right\}=\frac{1}{2}\times\left(-1+0+2-1\right)=0$$

$a_0 = 0$

d.

$$a_k=\frac{1}{T_0}\int_0^{T_0}x(t)e^{-j(2\pi/T_0)kt}dt=\frac{1}{2}\left\{\int_0^1(-1)e^{-j(2\pi/2)kt}dt+\int_1^2 e^{-j(2\pi/2)kt}dt\right\}$$

$$a_k=\frac{1}{2}\times\left\{\frac{-e^{-j(2\pi/2)kt}}{-j(2\pi/2)k}\Bigg|_0^1+\frac{e^{-j(2\pi/2)kt}}{-j(2\pi/2)k}\Bigg|_1^2\right\}=\frac{j}{2\pi k}\times\left(-e^{-j\pi k}+1+e^{-j2\pi k}-e^{-j\pi k}\right)$$

$$a_k=\frac{j(1+e^{-j2\pi k}-2e^{-j\pi k})}{2\pi k}=\frac{j(1+1-2e^{-j\pi k})}{2\pi k}=\frac{j(2-2(-1)^k)}{2\pi k}=\frac{j(1-(-1)^k)}{\pi k}$$

$$\boldsymbol{a_k=\frac{j(1-(-1)^k)}{\pi k}}$$

e.

for $k = 1$; $a_1=\frac{j(1-(-1)^1)}{\pi}=\frac{j(1+1)}{\pi}=\frac{j2}{\pi}$ for $k = -1$; $a_{-1}=\frac{j(1-(-1)^{-1})}{-\pi}=\frac{-j(1+1)}{\pi}=\frac{-j2}{\pi}$

for $k = 2$; $a_2=\frac{j(1-(-1)^2)}{\pi 2}=\frac{j(1-1)}{2\pi}=0$ for $k = -2$; $a_{-2}=\frac{j(1-(-1)^{-2})}{-\pi 2}=\frac{-j(1-1)}{2\pi}=0$

for $k = 3$; $a_3=\frac{j(1-(-1)^3)}{\pi 3}=\frac{j(1+1)}{3\pi}=\frac{j2}{3\pi}$ for $k = -3$; $a_{-3}=\frac{j(1-(-1)^{-3})}{-\pi 3}=\frac{-j(1+1)}{3\pi}=\frac{-j2}{3\pi}$