Midterm (20.11.2018) (13:00-15:00)

BLM 2401- Signals and Systems

Q1(10)	Q2(10)	Q3(15)	Q4(10)	Q5(15)	Q6(15)	Q7(15)	Q8(15)	Q9(00)	Q10(00)	Total

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 \qquad E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

Q01. Determine whether the following statements are correct or not by placing **F**(alse) or **T**(rue) in parenthesis. (10)

a. An input to the LTI system of $\alpha x_1(t)$ produces output $\alpha y_1(t)$. (T)

b. An input to the LTI system of $x_1(t) + x_2(t)$ produces output $y_1(t) + y_2(t)$. (T)

c. An input to the LTI system of $x_1(t - t_0) + x_2(t - t_1)$ produces output $y_1(t - t_1) + y_2(t - t_0)$, where $t_0 \neq t_1$. (F)

d. Normalized frequency (f/f_s) of a digital signal cannot be greater than 0.5. (T)

e. Periodic signals can only have: $f_k = kf_0$. (T)

f. A video is a two dimensional signal. (F)

g. If x(t) is an energy signal, then the average power P = 0. (T)

h. A periodic signal is an energy signal. (F)

i. The product of an even function and an odd function is odd. (T)

j. Any real time-dependent system is causal. (T)

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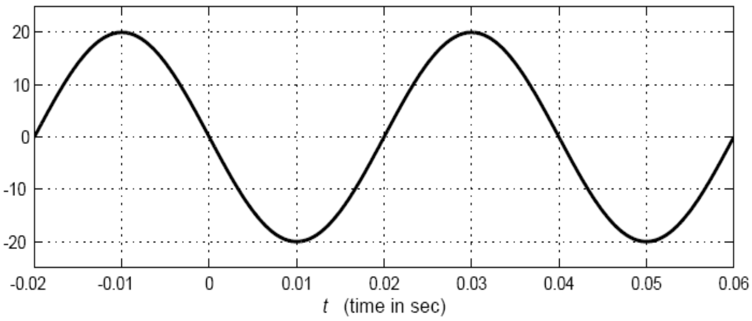
Q02. Given the following waveform;

a. Write corresponding sinusoidal equation (in terms of the cosine, $A\cos(\omega t + \phi)$). (05)



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a. $A = 20 \qquad T = 0.04\,\text{s} \qquad f = \frac{1}{T} = \frac{1}{0.04} = 25\ \text{Hz} \qquad \phi = \frac{\pi}{2}$

$x(t) = A\cos(\omega t + \phi) = 20\cos(2\times 25\pi t + \frac{\pi}{2}) = 20\cos(50\pi t + \frac{\pi}{2})$

b. Because f is 25 Hz, minimum sampling frequency must be 50 Hz.

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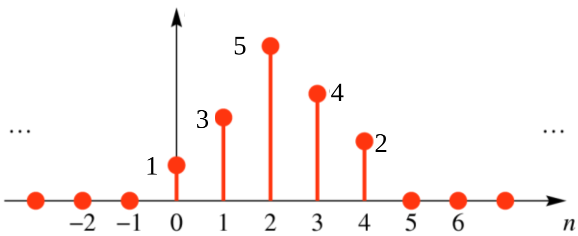
Q03. Discrete signal (or samples in a data file) $x[n]$ is given by the following graph.

a. Plot the filtered output signal $y[n]$ when a causal two point moving average filter $((x[n-1]+ x[n])/2)$ is applied to $x[n]$. (08)



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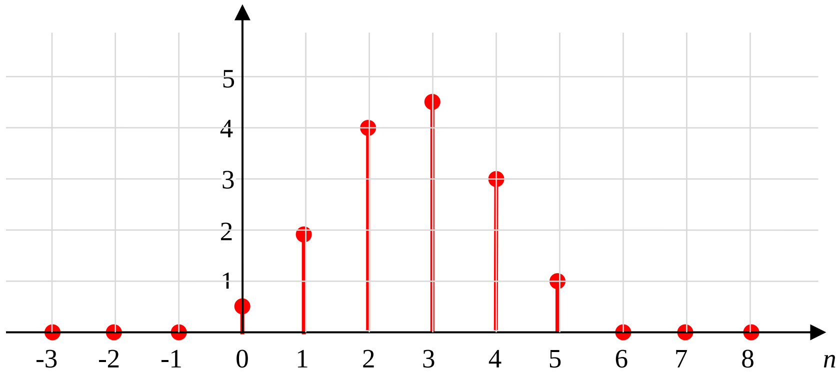
a.



b. Prove that convolution of $x[n]$ and impulse response of the filter $h[n]=0.5\delta[n-1]+0.5\delta[n]$ gives the same result as (a). (07)

{bk} = {0.5, 0.5}; {x[n]} = {1, 3, 5, 4, 2} y[n] = h[n] * x[n] = x[n] * h[n]

$x[n]$	1	3	5	4	2	
$h[n]$	0.5	0.5				
	0.5	1.5	2.5	2	1	
		0.5	1.5	2.5	2	1
$y[n]$	0.5	2	4	4.5	3	1

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Q04. Find the even and odd components of the following signal: (10)

$$x(t) = 1 + t\cos(t) + t^2\sin(t) + t^3\sin(t)\cos(t)$$

Even: $1 + t^3\sin(t)\cos(t)$
Odd: $t\cos(t) + t^2\sin(t)$

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Q05. Consider the discrete time signal: $x[n] = \begin{cases} n, & 0 \le n \le 5 \\ 10-n, & 5 < n \le 10 \\ 0, & otherwise \end{cases}$

a. Determine the energy and power of $x[n]$. (09)

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$
$$= \sum_{n=0}^{5} n^2 + \sum_{n=6}^{10} (10-n)^2$$
$$= (1^2+2^2+3^2+4^2+5^2) + (4^2+3^2+2^2+1^2) = 85$$

b. Is it an energy-signal or a power-signal? (06)

Since the signal $x[n]$ is an energy signal, its power is zero over $(-\infty, \infty)$.
The energy is finite so the signal $x[n]$ is an energy signal.

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Q06. Simplify the following expressions and write them in Cartesian form: (15)

a. $3e^{j\pi/3} + 4e^{-j\pi/6} =$

$$3e^{j\pi/3} + 4e^{-j\pi/6} = 3cos\frac{\pi}{3} + j3sin\frac{\pi}{3} + 4cos\frac{\pi}{6} - j4sin\frac{\pi}{6} = 2.598 + 3.8637 + j(1.5 - 1.035) \cong 6.4 + j0.5$$

b. $j^j =$

$$j^j = \exp\left(j\frac{\pi}{2}\right)^j = \exp\left(j^2\frac{\pi}{2}\right) = \exp\left(-\frac{\pi}{2}\right)$$

c. $(1+j)^j =$

$$(1+j)^j = \left(\sqrt{2}\exp\left(\frac{j\pi}{4}\right)\right)^j = \exp\left(j^2\frac{\pi}{4}\right)\left(\sqrt{2}\right)^j = \exp\left(-\frac{\pi}{4}\right)e^{j\log(\sqrt{2})}$$
$$= \exp\left(-\frac{\pi}{4}\right)\cos(\log(\sqrt{2})) + j\exp\left(-\frac{\pi}{4}\right)\sin(\log(\sqrt{2}))$$

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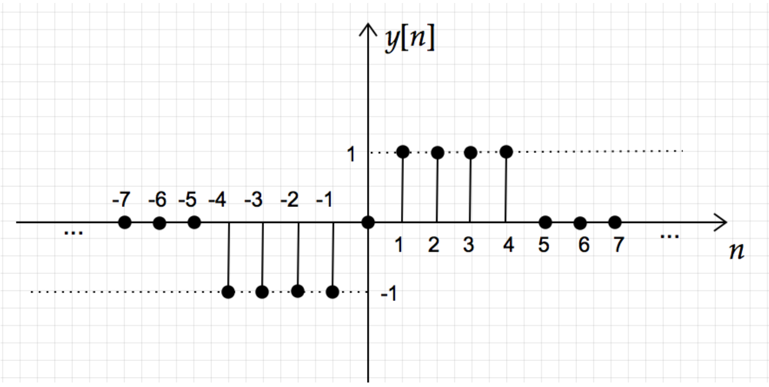
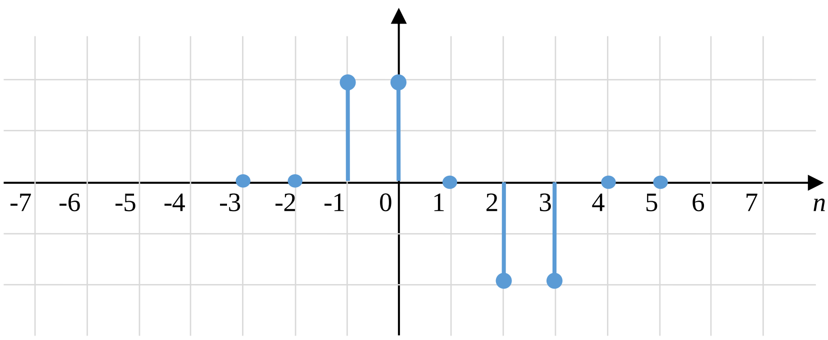
Q7. Let $y[n]$ be given in the following figure.



a. Express $y[n]$ in terms of DT unit step signal $u[n] = \begin{cases}1, & n \geq 0\\ 0, & \text{otherwise}\end{cases}$ (08)

$$y[n] = -u[n+4] + u[n] + u[n-1] - u[n-5]$$

b. Carefully sketch the signal $y[2-2n]$. (07)



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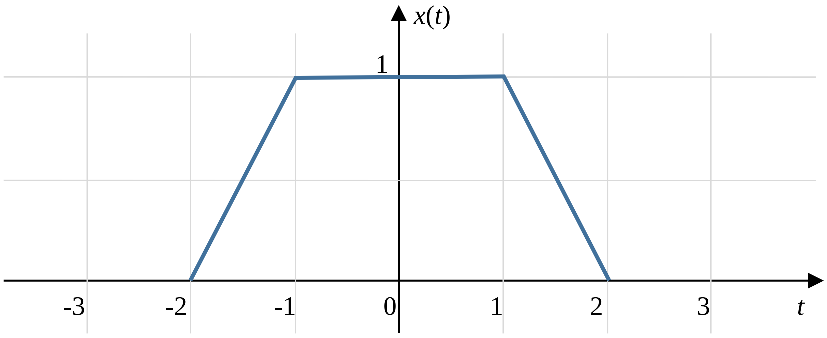
Q08. Consider a trapezoidal pulse, which is defined by $x(t) = \begin{cases} 2+t & -2 \le t \le -1 \\ 1 & -1 \le t \le 1 \\ 2-t & 1 \le t \le 2 \end{cases}$

a. Sketch the signal. (07)



b. Find the energy content of this signal. (08)

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E = \int_{-2}^{-1} |2+t|^2 dt + \int_{-1}^{1} |1|^2 dt + \int_{1}^{2} |2-t|^2 dt$$

Using the following substitutions;

2 + *t* = u; dt = du when *t* = -2 → *u* = 0 when *t* = -1 → *u* = 1
2 - *t* = v; dt = -dv when *t* = 1 → *v* = 1 when *t* = 2 → *v* = 0

$$E = \int_{0}^{1} |u|^2 du + \int_{-1}^{1} |1|^2 dt - \int_{1}^{0} |v|^2 dv = \frac{u^3}{3}\Big|_0^1 + t|_{-1}^{1} - \frac{v^3}{3}\Big|_1^0 = \frac{1}{3} + 2 + \frac{1}{3} = \frac{8}{3} \cong 2.67$$

Or;

$$E = \frac{(2+t)^3}{3}\Big|_{-2}^{-1} + t|_{-1}^{1} - \frac{(2-t)^3}{3}\Big|_1^2 = \frac{1}{3} + 2 + \frac{1}{3} = \frac{8}{3} \cong 2.67$$

Or using the symmetry and the following substitutions;

2 - *t* = v; dt = -dv when *t* = 1 → *v* = 1 when *t* = 2 → *v* = 0

$$E = 2\int_{0}^{1} |1|^2 dt + 2\int_{1}^{2} |2-t|^2 dt = 2\int_{0}^{1} dt - 2\int_{1}^{0} |v|^2 dv = 2\left(t|_0^1 - \frac{v^3}{3}\Big|_1^0\right) = 2\left(1 + \frac{1}{3}\right) = \frac{8}{3} \cong 2.67$$