

Electronic Circuits

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BJT Transistor Modeling

- A model is an equivalent circuit that represents the AC characteristics of the transistor.
- A model uses circuit elements that approximate the behavior of the transistor.
- There are two models commonly used in small signal AC analysis of a transistor:
 - r_e model
 - Hybrid equivalent model

The r_e Transistor Model

- BJTs are basically **current-controlled** devices; therefore the r_e model uses a diode and a current source to duplicate the behavior of the transistor.
- One disadvantage to this model is its sensitivity to the DC level. This model is designed for specific circuit conditions.

Common-Base Configuration

$$I_e = \alpha I_c \quad r_e = \frac{26 \text{ mV}}{I_e}$$

Input impedance:

$$Z_i = r_e$$

Output impedance:

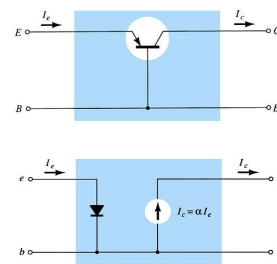
$$Z_o \cong \infty \Omega$$

Voltage gain:

$$A_V = \frac{\alpha R_L}{r_e} \cong \frac{R_L}{r_e}$$

Current gain:

$$A_i = -\alpha \cong -1$$

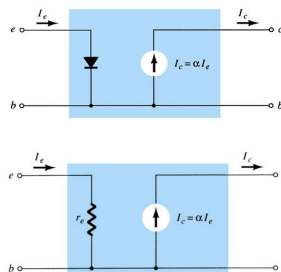


Common-Emitter Configuration

The diode r_e model can be replaced by the resistor r_e .

$$I_e = (\beta + 1)I_b \cong \beta I_b$$

$$r_e = \frac{26 \text{ mV}}{I_e}$$



more...

Common-Emitter Configuration

Input impedance:

$$Z_i = \beta r_e$$

Output impedance:

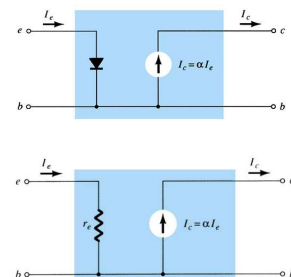
$$Z_o = r_o \cong \infty \Omega$$

Voltage gain:

$$A_V = -\frac{R_L}{r_e}$$

Current gain:

$$A_i = \beta \Big|_{r_o = \infty}$$



Common-Collector Configuration

Input impedance:

$$Z_i = (\beta + 1)r_e$$

Output impedance:

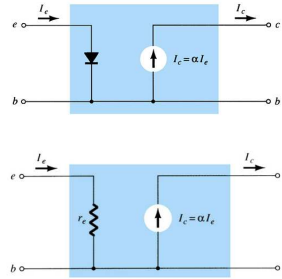
$$Z_o = r_e \parallel R_E$$

Voltage gain:

$$A_v = \frac{R_E}{R_E + r_e}$$

Current gain:

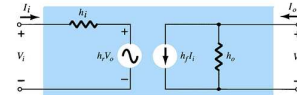
$$A_i = \beta + 1$$



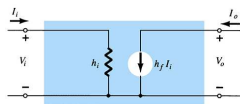
The Hybrid Equivalent Model

The following hybrid parameters are developed and used for modeling the transistor. These parameters can be found on the specification sheet for a transistor.

- h_i = input resistance
- h_r = reverse transfer voltage ratio (V_i/V_o) $\cong 0$
- h_f = forward transfer current ratio (I_o/I_i)
- h_o = output conductance



Simplified General h-Parameter Model



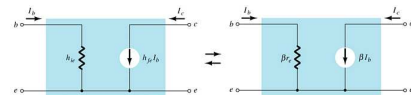
- h_i = input resistance
- h_f = forward transfer current ratio (I_o/I_i)

r_e vs. h-Parameter Model

Common-Emitter

$$h_{ie} = \beta r_e$$

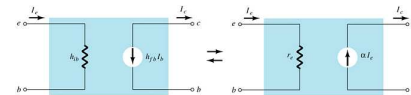
$$h_{fe} = \beta_{ac}$$



Common-Base

$$h_{ib} = r_e$$

$$h_{fb} = -\alpha \cong -1$$



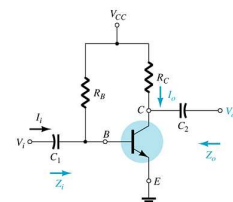
The Hybrid π Model

The hybrid π model is most useful for analysis of high-frequency transistor applications.

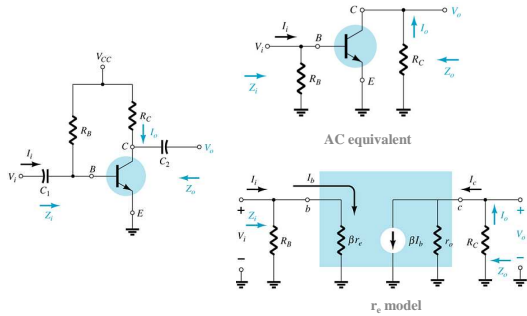
At lower frequencies the hybrid π model closely approximate the r_e parameters, and can be replaced by them.

Common-Emitter Fixed-Bias Configuration

- The input is applied to the base
 - High input impedance
 - Low output impedance
- High voltage and current gain
- Phase shift between input and output is 180°



Common-Emitter Fixed-Bias Configuration



Common-Emitter Fixed-Bias Calculations

Input impedance:

$$Z_i = R_B \parallel \beta r_e$$

$$Z_i \approx \beta r_e \quad R_B \geq 10\beta r_e$$

Output impedance:

$$Z_o = R_C \parallel r_o$$

$$Z_o \approx R_C \quad r_o \geq 10R_C$$

Voltage gain:

$$A_v = \frac{V_o}{V_i} = -\frac{(R_C \parallel r_o)}{r_e}$$

$$A_v \approx -\frac{R_C}{r_e} \quad r_o \geq 10R_C$$

Current gain:

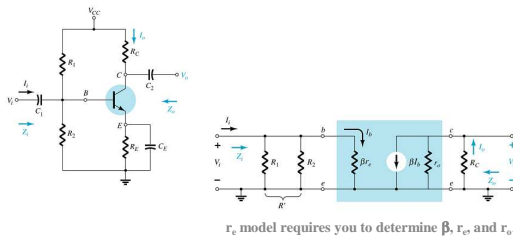
$$A_i = \frac{I_o}{I_i} = \frac{\beta R_B r_o}{(r_o + R_C)(R_B + \beta r_e)}$$

$$A_i \approx \beta \quad r_o \geq 10R_C, R_B \geq 10\beta r_e$$

Current gain from voltage gain:

$$A_i = -A_v \frac{Z_i}{R_C}$$

Common-Emitter Voltage-Divider Bias



Common-Emitter Voltage-Divider Bias Calculations

Input impedance:

$$R' = R_1 \parallel R_2$$

$$Z_i = R' \parallel \beta r_e$$

Output impedance:

$$Z_o = R_C \parallel r_o$$

$$Z_o \approx R_C \quad r_o \geq 10R_C$$

Voltage gain:

$$A_v = \frac{V_o}{V_i} = -\frac{R_C \parallel r_o}{r_e}$$

$$A_v \approx -\frac{R_C}{r_e} \quad r_o \geq 10R_C$$

Current gain:

$$A_i = \frac{I_o}{I_i} = \frac{\beta R' r_o}{(r_o + R_C)(R' + \beta r_e)}$$

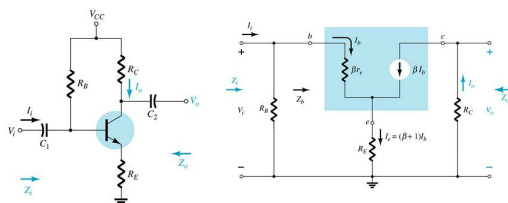
$$A_i \approx \frac{I_o}{I_i} = \frac{\beta R'}{R' + \beta r_e} \quad r_o \geq 10R_C$$

$$A_i \approx \beta \quad r_o \geq 10R_C, R' \geq 10\beta r_e$$

Current gain from voltage gain:

$$A_i = -A_v \frac{Z_i}{R_C}$$

Common-Emitter Emitter-Bias Configuration



Impedance Calculations

Input impedance:

$$Z_i = R_B \parallel Z_b$$

$$Z_b = \beta r_e + (\beta + 1)R_E$$

$$Z_b \approx \beta(r_e + R_E)$$

$$Z_b \approx \beta R_E$$

Output impedance:

$$Z_o = R_C$$

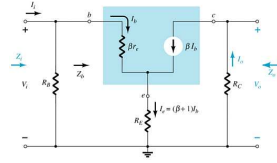
Gain Calculations

Voltage gain:

$$A_v = \frac{V_o}{V_i} = -\frac{\beta R_C}{Z_b}$$

$$A_v = \frac{V_o}{V_i} = -\frac{R_C}{r_e + R_E} \quad | \quad Z_b = \beta(r_e + R_E)$$

$$A_v = \frac{V_o}{V_i} = -\frac{R_C}{R_E} \quad | \quad Z_b = \beta R_E$$



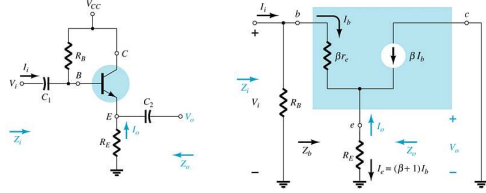
Current gain:

$$A_i = \frac{I_o}{I_i} = \frac{\beta R_B}{R_B + Z_b}$$

Current gain from voltage gain:

$$A_i = -A_v \frac{Z_i}{R_C}$$

Emitter-Follower Configuration



- This is also known as the common-collector configuration.
- The input is applied to the base and the output is taken from the emitter.
- There is no phase shift between input and output.

Impedance Calculations

Input impedance:

$$Z_i = R_B \parallel Z_b$$

$$Z_b = \beta r_e + (\beta + 1)R_E$$

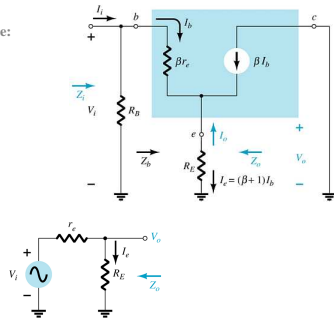
$$Z_b \approx \beta(r_e + R_E)$$

$$Z_b \approx \beta R_E$$

Output impedance:

$$Z_o = R_E \parallel r_e$$

$$Z_o \approx r_e \quad | \quad R_E \gg r_e$$



Gain Calculations

Voltage gain:

$$A_v = \frac{V_o}{V_i} = \frac{R_E}{R_E + r_e}$$

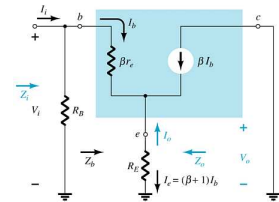
$$A_v = \frac{V_o}{V_i} \approx 1 \quad | \quad R_E \gg r_e, R_E + r_e \approx R_E$$

Current gain:

$$A_i = \frac{\beta R_B}{R_B + Z_b}$$

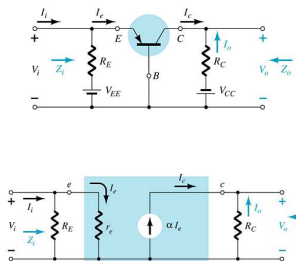
Current gain from voltage gain:

$$A_i = -A_v \frac{Z_i}{R_E}$$



Common-Base Configuration

- The input is applied to the emitter.
- The output is taken from the collector.
- Low input impedance.
- High output impedance.
- Current gain less than unity.
- Very high voltage gain.
- No phase shift between input and output.



Input impedance:

$$Z_i = R_E \parallel r_e$$

Output impedance:

$$Z_o = R_C$$

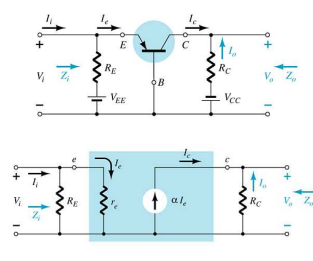
Voltage gain:

$$A_v = \frac{V_o}{V_i} = \frac{\alpha R_C}{r_e} \approx \frac{R_C}{r_e}$$

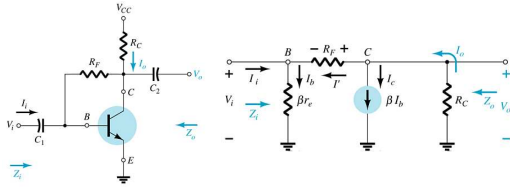
Current gain:

$$A_i = \frac{I_o}{I_i} = -\alpha \approx -1$$

Calculations



Common-Emitter Collector Feedback Configuration



- This is a variation of the common-emitter fixed-bias configuration
 - Input is applied to the base
 - Output is taken from the collector
- There is a 180° phase shift between input and output

Calculations

Input impedance:

$$Z_i = \frac{r_e}{\beta} \parallel R_C \parallel R_F$$

Output impedance:

$$Z_o \approx R_C \parallel R_F$$

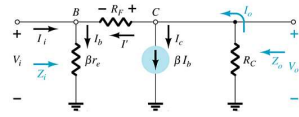
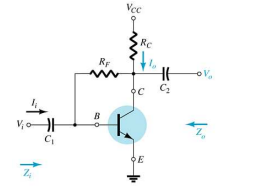
Voltage gain:

$$A_v = \frac{V_o}{V_i} = -\frac{R_C}{r_e}$$

Current gain:

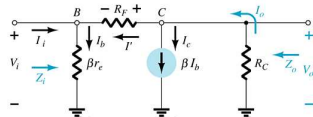
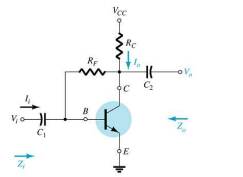
$$A_i = \frac{I_o}{I_i} = \frac{\beta R_F}{R_F + \beta R_C}$$

$$A_i = \frac{I_o}{I_i} \approx \frac{R_F}{R_C}$$



Collector DC Feedback Configuration

- This is a variation of the common-emitter, fixed-bias configuration
- The input is applied to the base
- The output is taken from the collector
- There is a 180° phase shift between input and output



Calculations

Input impedance:

$$Z_i = \frac{r_e}{\beta} \parallel R_C \parallel R_F$$

Output impedance:

$$Z_o \approx R_C \parallel R_F$$

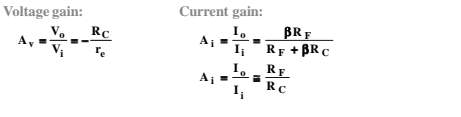
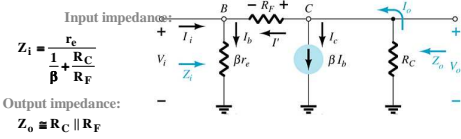
Voltage gain:

$$A_v = \frac{V_o}{V_i} = -\frac{R_C}{r_e}$$

Current gain:

$$A_i = \frac{I_o}{I_i} = \frac{\beta R_F}{R_F + \beta R_C}$$

$$A_i = \frac{I_o}{I_i} \approx \frac{R_F}{R_C}$$



Two-Port Systems Approach

This approach:

- Reduces a circuit to a two-port system
- Provides a "Thévenin look" at the output terminals
- Makes it easier to determine the effects of a changing load

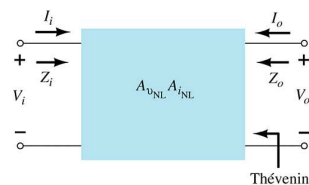
With V_i set to 0 V:

$$Z_{Th} = Z_o = R_o$$

The voltage across the open terminals is:

$$E_{Th} = A_{vNL} V_i$$

where A_{vNL} is the no-load voltage gain.



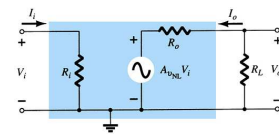
Effect of Load Impedance on Gain

This model can be applied to any current- or voltage-controlled amplifier.

Adding a load reduces the gain of the amplifier:

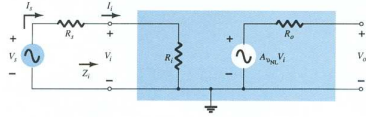
$$A_v = \frac{V_o}{V_i} = \frac{R_L}{R_L + R_o} A_{vNL}$$

$$A_i = -A_v \frac{Z_i}{R_L}$$



Effect of Source Impedance on Gain

The fraction of applied signal that reaches the input of the amplifier is:

$$V_i = \frac{R_i V_s}{R_i + R_s}$$


The internal resistance of the signal source reduces the overall gain:

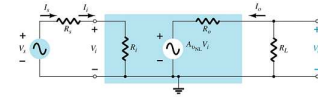
$$A_{vs} = \frac{V_o}{V_s} = \frac{R_i}{R_i + R_s} A_{vNL}$$

Combined Effects of R_s and R_L on Voltage Gain

Effects of R_L :

$$A_v = \frac{V_o}{V_i} = \frac{R_L A_{vNL}}{R_L + R_o}$$

$$A_i = -A_v \frac{R_i}{R_L}$$



Effects of R_L and R_s :

$$A_{vs} = \frac{V_o}{V_s} = \frac{R_i}{R_i + R_s} \frac{R_L}{R_L + R_o} A_{vNL}$$

$$A_{is} = -A_{vs} \frac{R_s + R_i}{R_L}$$

Cascaded Systems

- The output of one amplifier is the input to the next amplifier
- The overall voltage gain is determined by the product of gains of the individual stages
- The DC bias circuits are isolated from each other by the coupling capacitors
- The DC calculations are independent of the cascading
- The AC calculations for gain and impedance are interdependent

R-C Coupled BJT Amplifiers

Input impedance, first stage:

$$Z_i = R_1 \parallel R_2 \parallel \beta r_e$$

Output impedance, second stage:

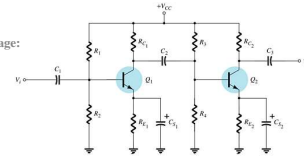
$$Z_o = R_C$$

Voltage gain:

$$A_{v1} = \frac{R_C \parallel R_1 \parallel R_2 \parallel \beta r_e}{r_e}$$

$$A_{v2} = \frac{R_C}{r_e}$$

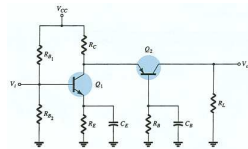
$$A_v = A_{v1} A_{v2}$$



Cascode Connection

This example is a CE-CB combination. This arrangement provides high input impedance but a low voltage gain.

The low voltage gain of the input stage reduces the Miller input capacitance, making this combination suitable for high-frequency applications.

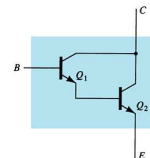


Darlington Connection

The Darlington circuit provides a very high current gain—the product of the individual current gains:

$$\beta_D = \beta_1 \beta_2$$

The practical significance is that the circuit provides a very high input impedance.



DC Bias of Darlington Circuits

Base current:

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + \beta_D R_E}$$

Emitter current:

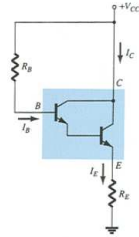
$$I_E = (\beta_D + 1)I_B \approx \beta_D I_B$$

Emitter voltage:

$$V_E = I_E R_E$$

Base voltage:

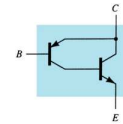
$$V_B = V_E + V_{BE}$$



Feedback Pair

This is a two-transistor circuit that operates like a Darlington pair, but it is not a Darlington pair.

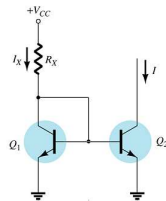
- It has similar characteristics:
- High current gain
 - Voltage gain near unity
 - Low output impedance
 - High input impedance



The difference is that a Darlington uses a pair of like transistors, whereas the feedback-pair configuration uses complementary transistors.

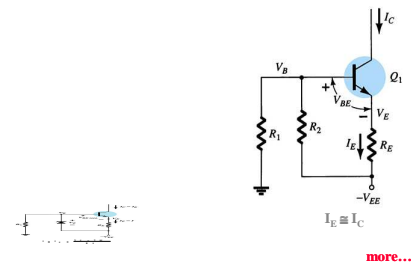
Current Mirror Circuits

Current mirror circuits provide constant current in integrated circuits.



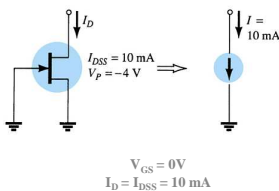
Current Source Circuits

Constant-current sources can be built using FETs, BJTs, and combinations of these devices.



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Current Source Circuits



Fixed-Bias Configuration

Input impedance:

$$Z_i = R_B \parallel h_{ie}$$

Output impedance:

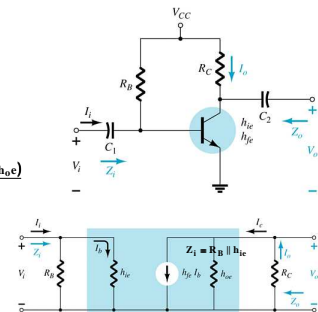
$$Z_o = R_C \parallel 1/h_{oe}$$

Voltage gain:

$$A_v = \frac{V_o}{V_i} = -\frac{h_{fe}(R_C \parallel 1/h_{oe})}{h_{ie}}$$

Current gain:

$$A_i = \frac{I_o}{I_i} \approx h_{fe}$$



Voltage-Divider Configuration

Input impedance:

$$Z_i = R'_1 \parallel h_{ie}$$

Output impedance:

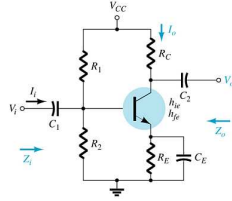
$$Z_o = R_C$$

Voltage gain:

$$A_v = -\frac{h_{fe}(R_C \parallel 1/h_{oc})}{h_{ie}}$$

Current gain:

$$A_i = -\frac{h_{fe}R'_1}{R'_1 + h_{ie}}$$



Emitter-Follower Configuration

Input impedance:

$$Z_b = h_{fe}R_E$$

$$Z_i = R_B \parallel Z_b$$

Output impedance:

$$Z_o = R_E \parallel \frac{h_{ie}}{h_{fe}}$$

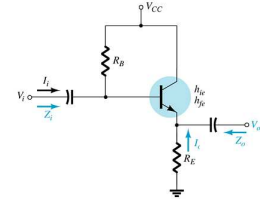
Voltage gain:

$$A_v = \frac{V_o}{V_i} = \frac{R_E}{R_E + h_{ie}/h_{fe}}$$

Current gain:

$$A_i = \frac{h_{fe}R_B}{R_B + Z_b}$$

$$A_i = -A_v \frac{Z_i}{R_E}$$



Common-Base Configuration

Input impedance:

$$Z_i = R_E \parallel h_{ib}$$

Output impedance:

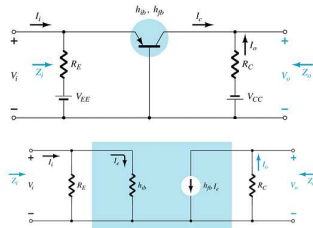
$$Z_o = R_C$$

Voltage gain:

$$A_v = \frac{V_o}{V_i} = -\frac{h_{fb}R_C}{h_{ib}}$$

Current gain:

$$A_i = \frac{I_o}{I_i} = h_{fb} \approx -1$$



Troubleshooting

Check the DC bias voltages

- ✓ If not correct, check power supply, resistors, transistor. Also check the coupling capacitor between amplifier stages.

Check the AC voltages

- ✓ If not correct check transistor, capacitors and the loading effect of the next stage.