Digital Signal Processing

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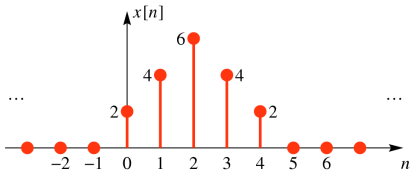
Q13

Determine the output of a *centralized averager*

$y[n]=(1/3)\,(x[n+1]+x[n]+x[n-1])$

for the following input. Is this filter causal or noncausal?

What is the support of the output for this input?



A13

$y[0] = \frac{1}{3}(x[1]+x[0]+x[-1]) = \frac{1}{3}(4+2+0) = 2$

$y[-1] = \frac{1}{3}(x[0]+x[-1]+x[-2]) = \frac{1}{3}(2+0+0) = 2/3$

$y[-2] = 0$

$y[1] = \frac{1}{3}(x[2]+x[1]+x[0]) = \frac{1}{3}(6+4+2) = 4$

Make a table:

n	≤-2	-1	0	1	2	3	4	5	6	≥7
x[n]	0	0	2	4	6	4	2	0	0	0
y[n]	0	2/3	2	4	14/3	4	2	2/3	0	0

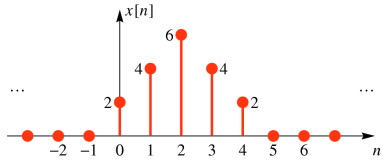
Since y[n] starts before x[n] ⇒ NOT causal

SUPPORT IS: $-1 \le n \le 5$

Q14

Compute the output $y[n]$ for the length-4 filter whose coefficients are $\{b_k\}=\{3, -1, 2, 1\}$. Use the the following signal as input. Verify that the answers tabulated here are correct, then fill in the missing values.

n	$n<0$	0	1	2	3	4	5	6	7	8	$n>8$
$x[n]$	0	2	4	6	4	2	0	0	0	0	0
$y[n]$	0	6	10	18	?	?	?	8	2	0	0



A14

$y[n] = \underset{b_0}{3}x[n] - \underset{b_1}{x[n-1]} + \underset{b_2}{2}x[n-2] + \underset{b_3}{x[n-3]}$

$y[2] = 3x[2] - x[1] + 2x[0] + x[-1]$

$= 3(6) - 4 + 2(2) + 0 = 18$ ✓

$y[3] = 3x[3] - x[2] + 2x[1] + x[0]$

$= 3(4) - 6 + 2(4) + 2 = \boxed{16}$

$y[4] = 3(2) - 4 + 2(6) + 4 = \boxed{18}$

$y[5] = 3(0) - 2 + 2(4) + 6 = \boxed{12}$

$y[6] = 3(0) - 0 + 2(2) + 4 = 8$ ✓

Q15

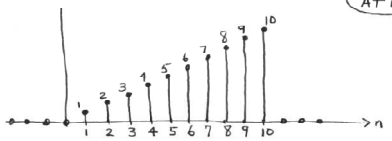
Determine and plot the impulse response of the FIR system

$$y[n] = \sum_{k=0}^{10} k x[n-k]$$

A15

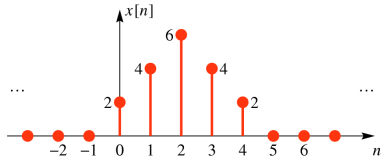
$$y[n] = \sum_{k=0}^{10} k\,\delta[n-k] = 0\cdot\delta[n] + \delta[n-1] + 2\,\delta[n-2] + \dots + 10\,\delta[n-10]$$

IMPULSE AT $n=2$



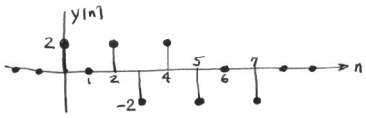
Q16

Use the "synthetic multiplication" convolution algorithm to compute the output $y[n[$ for the length 4 filter whose coefficients are $\{b_k\} = \{1, -2, 2, -1\}$. Use the input signal given below.



A16

$$\begin{array}{rrrrrrrrl}
2 & 4 & 6 & 4 & 2 & & & & \leftarrow x[n] \\
1 & -2 & 2 & -1 & & & & & \leftarrow b_k \\
\hline
2 & 4 & 6 & 4 & 2 & & & & \\
 & -4 & -8 & -12 & -8 & -4 & & & \\
 & & 4 & 8 & 12 & 8 & 4 & & \\
 & & & -2 & -4 & -6 & -4 & -2 & \\
\hline
2 & 0 & 2 & -2 & 2 & -2 & 0 & -2 & \leftarrow y[n] \\
\uparrow & & & & & \uparrow & & & \\
n=0 & & & & & n=5 & & &
\end{array}$$



Q17

In MATLAB, we can compute only the convolution of finite-length signals. Determine the length of the output sequence computed by the MATLAB convolution below.

```
xx = sin(0.07*pi*(0:50));
hh = ones(11,1)/11;
yy = conv(hh, xx);
```


A17

Length of yy = Length(xx) + Length(hh) − 1

Length of xx = 51 because 0:50 generates 0,1,2,…50

Length of hh = 11 ones(11,1) is 11-element column

⇒ Length of yy = 51 + 11 − 1 = 61

Q18

Use MATLAB to compute the following product of polynomials:

$$P(x) = (1 + 2x + 3x^2 + 5x^4)(1 - 3x - x^2 + x^3 + 3x^4)$$

A18

In MATLAB, use conv( ) function

conv([1, 2, 3, 0, 5], [1, -3, -1, 1, 3])

MISSING x^3 TERM

The answer computed by MATLAB is:

[1, -1, -4, -10, 7, -6, 4, 5, 15]

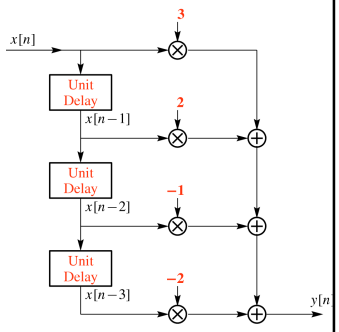
As a polynomial:

$$1 - x - 4x^2 - 10x^3 + 7x^4 - 6x^5 + 4x^6 + 5x^7 + 15x^8$$

13

Q19

Determine the difference equation for the following block diagram



14

A19

The outputs of the UNIT-DELAYS are already labelled, so each output is multiplied by a constant and then added to form y[n].

$$y[n] = -2x[n-3] + \left(-x[n-2] + \left(2x[n-1] + \left(3x[n]\right)\right)\right)$$

$$= 3x[n] + 2x[n-1] - x[n-2] - 2x[n-3]$$

The filter coefficients are:

$b_0 = 3$, $b_1 = 2$, $b_2 = -1$, and $b_3 = -2$

15

Q20

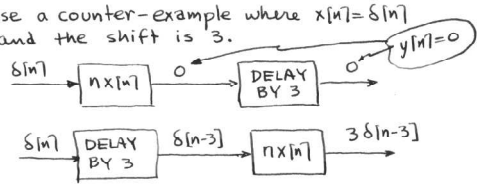
Test the system defined by the equation $y[n] = nx[n]$ to determine whether it is a time-invariant system.

16

A20

$y[n] = n\,x[n]$ is NOT time-invariant.

Use a counter-example where $x[n] = \delta[n]$ and the shift is 3.



The outputs are NOT equal so the system is NOT time-invariant

17

Q21

Show that the time-flip system $y[n] = x[-n]$ is a linear system.

18

A21

Define $\text{FLIP}\{x[n]\} = x[-n]$

Scaling: $\text{FLIP}\{\alpha x[n]\} = \alpha x[-n]$

That is, "doubling $x[n]$" will double its flipped version

Additive:

$$\text{FLIP}\{x_1[n] + x_2[n]\} = x_1[-n] + x_2[-n]$$
$$= \text{FLIP}\{x_1[n]\} + \text{FLIP}\{x_2[n]\}$$

19

Q22

By making the substitution $k = n - l$ in the following equation,

$$y[n] = \sum_{l=n-M}^{n} x[l]h[n-l]$$

show that $y[n]$ can also be expressed in the same form as

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

20

A22

Start with $y[n] = \sum_{\ell=n-M}^{n} h[n-\ell]\, x[\ell]$

With $k = n-\ell$, the limits on the sum are:

$\ell = n-M \implies k = n-\ell = n-(n-M) = M$

$\ell = n \implies k = n-\ell = n-n = 0$

$\therefore\ y[n] = \sum_{k=0}^{M} h[k]\, x[n-k]$

21

Q23

Determine the impulse response $h[n[$ of the 51-point causal running averager and determine the impulse response $\tilde{h}[n]$ for the 51-point centralized running averager.

22

A23 51-pt. CAUSAL RUNNING AVERAGE IS:

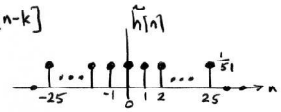
$$y[n] = \frac{1}{51}\sum_{k=0}^{50} x[n-k]$$

$$\Rightarrow h[n] = \frac{1}{51}\sum_{k=0}^{50} \delta[n-k]$$

51-pt CENTRAL RUNNING AVG:

$$\tilde{y}[n] = \frac{1}{51}\sum_{k=-25}^{25} x[n-k]$$

$$\Rightarrow \tilde{h}[n] = \frac{1}{51}\sum_{k=-25}^{25} \delta[n-k]$$



23