Digital Signal Processing

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Digital Signal Processing

Lecture 21

Amplitude Modulation

LECTURE OBJECTIVES

- Review of FT properties
  - Convolution <--> multiplication
  - Frequency shifting
- Sinewave Amplitude Modulation
  - AM radio
- Frequency-division multiplexing
  - FDM
- Reading: Chapter 12, Section 12-2

Table of Easy FT Properties

Linearity Property

$$ax_1(t)+bx_2(t) \Leftrightarrow aX_1(j\omega)+bX_2(j\omega)$$

Delay Property

$$x(t-t_d) \Leftrightarrow e^{-j\omega t_d}X(j\omega)$$

Frequency Shifting

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega-\omega_0))$$

Scaling

$$x(at) \Leftrightarrow \frac{1}{|a|}X(j(\frac{\omega}{a}))$$

Table of FT Properties

$$x(t)*h(t) \Leftrightarrow H(j\omega)X(j\omega)$$

$$x(t)p(t) \Leftrightarrow \frac{1}{2\pi}X(j\omega)*P(j\omega)$$

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega-\omega_0))$$

Differentiation Property

$$\frac{dx(t)}{dt} \Leftrightarrow (j\omega)X(j\omega)$$

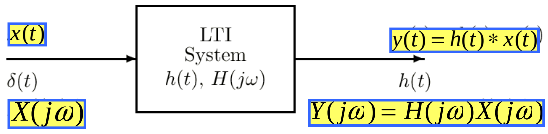
Frequency Shifting Property

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega-\omega_0))$$

$$\int_{-\infty}^{\infty} e^{-j\omega_0 t} x(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} x(t) e^{-j(\omega-\omega_0)t} dt$$
$$= X(j(\omega-\omega_0))$$

$$y(t) = \frac{\sin 7t}{\pi t} e^{j\omega_0 t} \Leftrightarrow Y(j\omega) = \begin{cases} 1 & \omega_0 - 7 < \omega < \omega_0 + 7 \\ 0 & elsewhere \end{cases}$$

Convolution Property



- Convolution in the time-domain

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

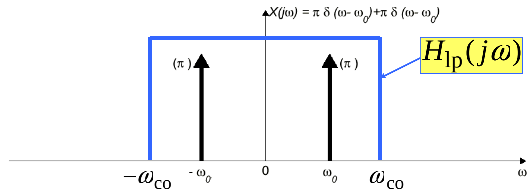
corresponds to **MULTIPLICATION** in the frequency-domain $Y(j\omega) = H(j\omega)X(j\omega)$

Cosine Input to LTI System

$$Y(j\omega) = H(j\omega)X(j\omega)$$
$$= H(j\omega)[\pi\delta(\omega-\omega_0) + \pi\delta(\omega+\omega_0)]$$
$$= H(j\omega_0)\pi\delta(\omega-\omega_0) + H(-j\omega_0)\pi\delta(\omega+\omega_0)$$

$$y(t) = H(j\omega_0)\tfrac{1}{2}e^{j\omega_0 t} + H(-j\omega_0)\tfrac{1}{2}e^{-j\omega_0 t}$$
$$= H(j\omega_0)\tfrac{1}{2}e^{j\omega_0 t} + H^*(j\omega_0)\tfrac{1}{2}e^{-j\omega_0 t}$$
$$= |H(j\omega_0)|\cos(\omega_0 t + \angle H(j\omega_0))$$

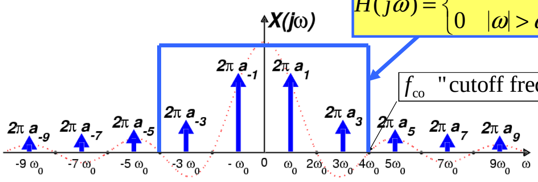
Ideal Lowpass Filter



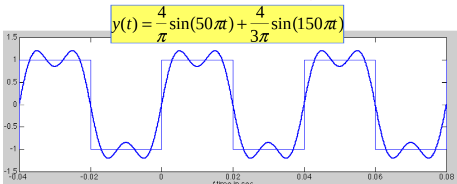
$$y(t) = x(t) \quad \text{if } \omega_0 < \omega_{co}$$
$$y(t) = 0 \quad \text{if } \omega_0 > \omega_{co}$$

Ideal LPF: Fourier Series

$$H(j\omega) = \begin{cases} 1 & |\omega| < \omega_{co} \\ 0 & |\omega| > \omega_{co} \end{cases}$$



$$y(t) = \frac{4}{\pi}\sin(50\pi t) + \frac{4}{3\pi}\sin(150\pi t)$$



The way communication systems work

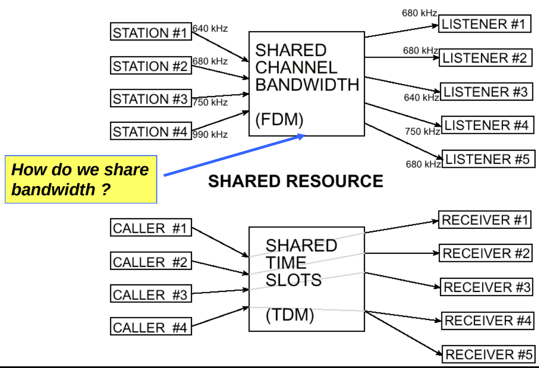


Table of FT Properties

$$x(t)*h(t) \Leftrightarrow H(j\omega)X(j\omega)$$

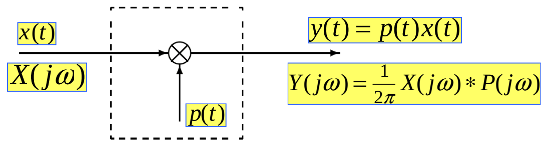
$$x(t)p(t) \Leftrightarrow \frac{1}{2\pi} X(j\omega)*P(j\omega)$$

$$x(t)e^{j\omega_0 t} \Leftrightarrow X(j(\omega-\omega_0))$$

Differentiation Property

$$\frac{dx(t)}{dt} \Leftrightarrow (j\omega)X(j\omega)$$

Signal Multiplier (Modulator)



- Multiplication in the time-domain corresponds to convolution in the frequency-domain.

$$Y(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)P(j(\omega-\theta))d\theta$$

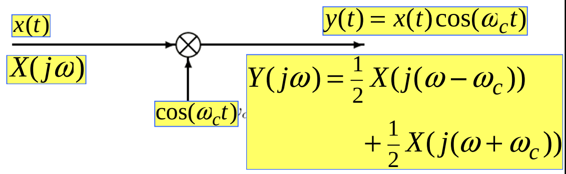
$$y(t) = x(t)p(t) \Leftrightarrow Y(j\omega) = \tfrac{1}{2\pi} X(j\omega)*P(j\omega)$$

$$p(t) = \cos(\omega_c t) \Leftrightarrow$$
$$P(j\omega) = \pi\delta(\omega-\omega_c) + \pi\delta(\omega+\omega_c)$$

$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow$$
$$Y(j\omega) = \tfrac{1}{2\pi} X(j\omega) * [\pi\delta(\omega-\omega_c) + \pi\delta(\omega+\omega_c)]$$

$$Y(j\omega) = \tfrac{1}{2} X(j(\omega-\omega_c)) + \tfrac{1}{2} X(j(\omega+\omega_c))$$

Amplitude Modulator



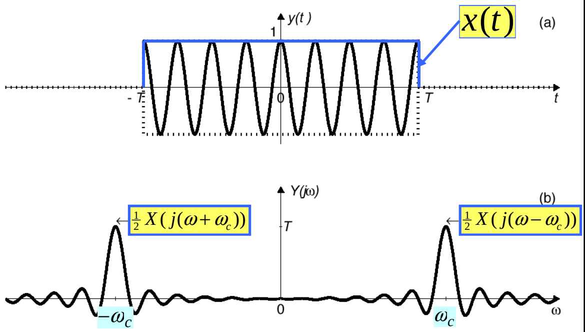
- $x(t)$ modulates the amplitude of the cosine wave. The result in the frequency-domain is two shifted copies of $X(j\omega)$.

$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow$$
$$Y(j\omega) = \tfrac{1}{2} X(j(\omega-\omega_c)) + \tfrac{1}{2} X(j(\omega+\omega_c))$$

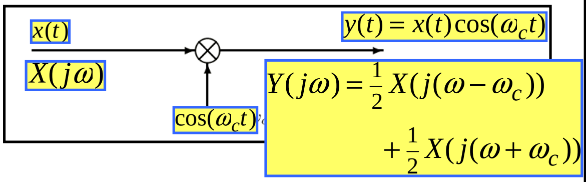
$$x(t) = \begin{cases} 1 & |t| < T \\ 0 & |t| > T \end{cases} \Leftrightarrow X(j\omega) = 2\frac{\sin(\omega T)}{(\omega)}$$

$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow$$
$$Y(j\omega) = \frac{\sin((\omega-\omega_c)T)}{(\omega-\omega_c)} + \frac{\sin((\omega+\omega_c)T)}{(\omega+\omega_c)}$$

$$y(t) = x(t)\cos(\omega_c t) \Leftrightarrow$$
$$Y(j\omega) = \tfrac{1}{2} X(j(\omega-\omega_c)) + \tfrac{1}{2} X(j(\omega+\omega_c))$$



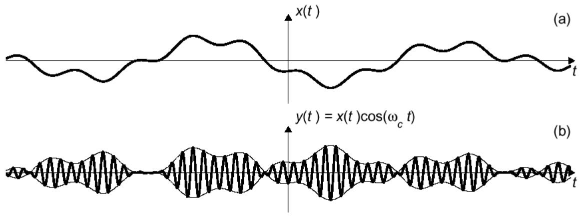
DSBAM Modulator



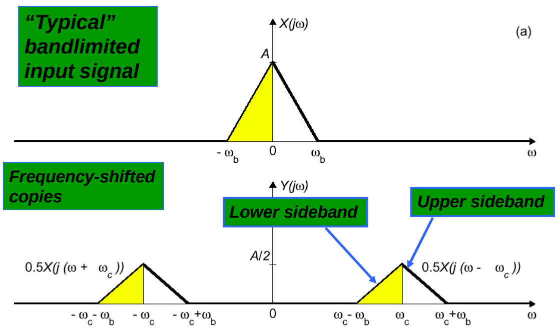
- If $X(j\omega)=0$ for $|\omega|>\omega_b$ and $\omega_c > \omega_b$, the result in the frequency-domain is two shifted and scaled **exact copies** of $X(j\omega)$.

DSBAM Waveform

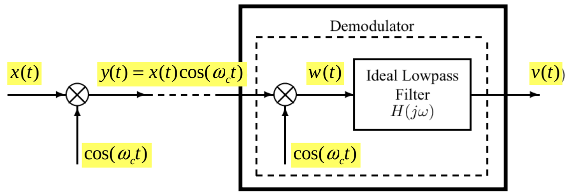
- In the time-domain, the "envelope" of sine-wave peaks follows $|x(t)|$



Double Sideband AM (DSBAM)



DSBAM DEmodulator

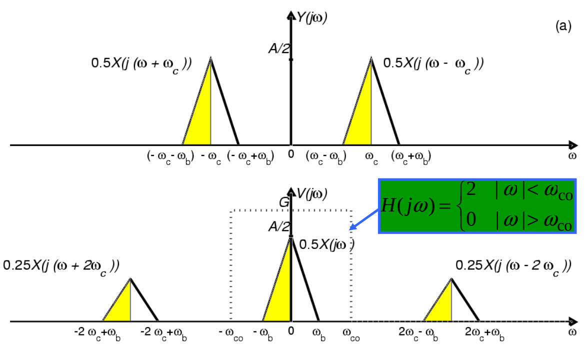


$$w(t) = x(t)[\cos(\omega_c t)]^2 = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2\omega_c t)$$

$$W(j\omega) = \frac{1}{2}X(j\omega) + \frac{1}{4}X(j(\omega - 2\omega_c)) + \frac{1}{4}X(j(\omega + 2\omega_c))$$

$$V(j\omega) = H(j\omega)W(j\omega)$$

DSBAM Demodulation

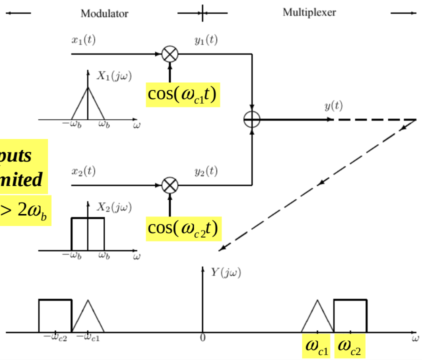


$$V(j\omega) = H(j\omega)W(j\omega) = X(j\omega) \text{ if } \omega_b < \omega_{co} < 2\omega_c - \omega_b$$

Frequency-Division Multiplexing (FDM)

- Shifting spectrum of signal to higher frequency:
  - Permits transmission of low-frequency signals with high-frequency EM waves
  - By allocating a frequency band to each signal multiple bandlimited signals can share the same channel
  - AM radio: 530-1620 kHz (10 kHz bands)
  - FM radio: 88.1-107.9 MHz (200 kHz bands)

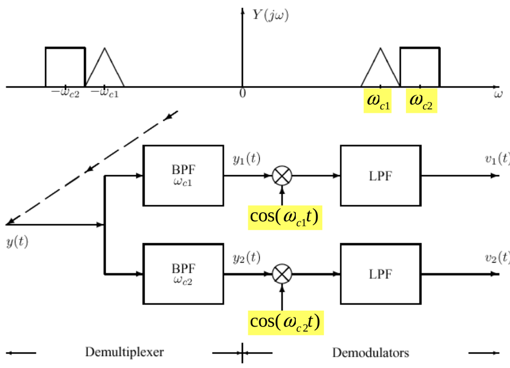
FDM Block Diagram (Xmitter)



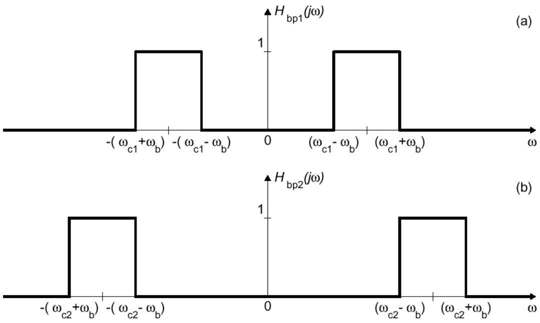
Spectrum of inputs must be bandlimited

Need $|\omega_{c2} - \omega_{c1}| > 2\omega_b$

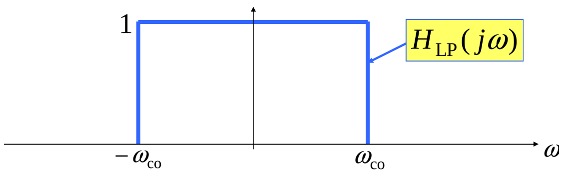
Frequency-Division De-Mux



Bandpass Filters for De-Mux



Pop Quiz: FT thru LPF



Input $x(t) \quad \leftrightarrow \quad X(j\omega) = \sum_{k=-\infty}^{\infty} 4\pi\delta(\omega - 30\pi k)$

If the output is $y(t) = 2$, then find a value for ω_{co}