

## Digital Signal Processing

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1

## Digital Signal Processing

Lecture 13

### Digital Filtering of Analog Signals

2

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3

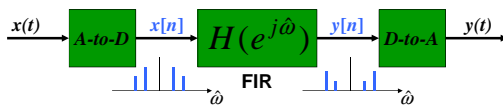
### READING ASSIGNMENTS

- This Lecture:
  - Chapter 6, Sections 6-6, 6-7 & 6-8
- Other Reading:
  - Recitation: Chapter 6
    - FREQUENCY RESPONSE EXAMPLES
  - Next Lecture: Chapter 7

4

### LECTURE OBJECTIVES

- Two Domains: Time & Frequency
- Track the spectrum of  $x[n]$  thru an FIR Filter:  
**Sinusoid-IN gives Sinusoid-OUT**
- **UNIFICATION**: How does Frequency Response affect  $x(t)$  to produce  $y(t)$  ?



### TIME & FREQUENCY

$$y[n] = \sum_{k=0}^M b_k x[n-k] = \sum_{k=0}^M h[k] x[n-k]$$

FIR DIFFERENCE EQUATION is the TIME-DOMAIN

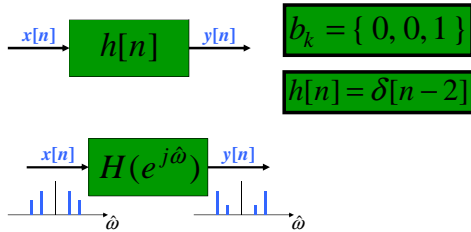
$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M h[k] e^{-j\hat{\omega}k}$$

$$H(e^{j\hat{\omega}}) = h[0] + h[1]e^{-j\hat{\omega}} + h[2]e^{-j2\hat{\omega}} + h[3]e^{-j3\hat{\omega}} + \dots$$

6

### Ex: DELAY by 2 SYSTEM

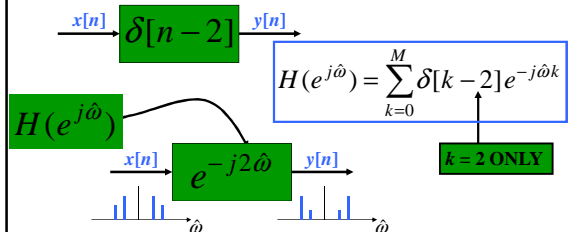
Find  $h[n]$  and  $H(e^{j\hat{\omega}})$  for  $y[n] = x[n - 2]$



7

### DELAY by 2 SYSTEM

Find  $h[n]$  and  $H(e^{j\hat{\omega}})$  for  $y[n] = x[n - 2]$



8

### GENERAL DELAY PROPERTY

Find  $h[n]$  and  $H(e^{j\hat{\omega}})$  for  $y[n] = x[n - n_d]$

$$h[n] = \delta[n - n_d]$$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M \delta[k - n_d] e^{-j\hat{\omega}k} = e^{-j\hat{\omega}n_d}$$

ONLY ONE non-ZERO TERM for  $k$  at  $k = n_d$

9

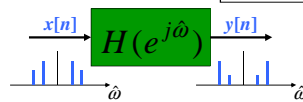
### FREQ DOMAIN --> TIME ??

• START with

$H(e^{j\hat{\omega}})$  and find  $h[n]$  or  $b_k$



$$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega})$$



10

### FREQ DOMAIN --> TIME

$$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega}) \quad \text{EULER's Formula}$$

$$= 7e^{-j2\hat{\omega}} (0.5e^{j\hat{\omega}} + 0.5e^{-j\hat{\omega}})$$

$$= (3.5e^{-j\hat{\omega}} + 3.5e^{-j3\hat{\omega}})$$

$$h[n] = 3.5\delta[n-1] + 3.5\delta[n-3]$$

$$b_k = \{ 0, 3.5, 0, 3.5 \}$$

11

### PREVIOUS LECTURE REVIEW

- SINUSOIDAL INPUT SIGNAL
  - OUTPUT has SAME FREQUENCY
  - DIFFERENT Amplitude and Phase
- FREQUENCY RESPONSE of FIR
  - MAGNITUDE vs. Frequency
  - PHASE vs. Freq
  - PLOTTING

$$H(e^{j\hat{\omega}}) = |H(e^{j\hat{\omega}})| e^{j\angle H(e^{j\hat{\omega}})}$$

MAG

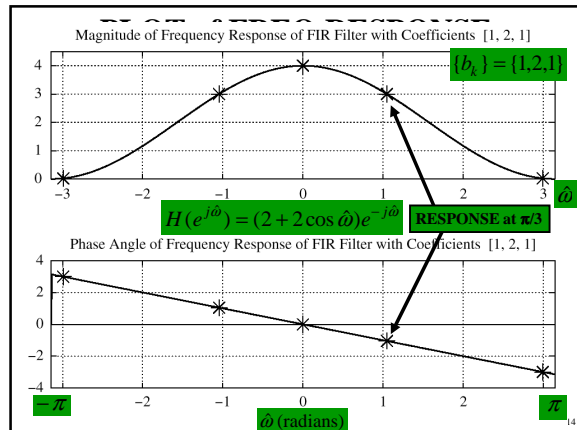
PHASE

12

## FREQ. RESPONSE PLOTS

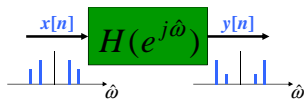
- DENSE GRID (**ww**) from  $-\pi$  to  $+\pi$   
 $-\text{ww} = -\text{pi}:(\text{pi}/100):\text{pi};$
- **HH** = **freqz(bb,1,ww)**  
 $-\text{VECTOR bb}$  contains Filter Coefficients  
 $-\text{DSP-First: HH} = \text{frekz(bb,1,ww)}$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^M b_k e^{-j\hat{\omega}k}$$



## EXAMPLE 6.2

Find  $y[n]$  when  $H(e^{j\hat{\omega}})$  is known  
 and  $x[n] = 2e^{j\pi/4} e^{j(\pi/3)n}$



$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega})e^{-j\hat{\omega}}$$

## EXAMPLE 6.2 (answer)

Find  $y[n]$  when  $x[n] = 2e^{j\pi/4} e^{j(\pi/3)n}$

One Step - evaluate  $H(e^{j\hat{\omega}})$  at  $\hat{\omega} = \pi/3$

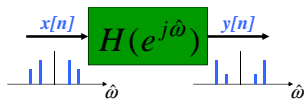
$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega})e^{-j\hat{\omega}}$$

$$H(e^{j\hat{\omega}}) = 3e^{-j\pi/3} \quad @ \hat{\omega} = \pi/3$$

$$y[n] = (3e^{-j\pi/3}) \times 2e^{j\pi/4} e^{j(\pi/3)n} = 6e^{-j\pi/12} e^{j(\pi/3)n}$$

## EXAMPLE: COSINE INPUT

Find  $y[n]$  when  $H(e^{j\hat{\omega}})$  is known  
 and  $x[n] = 2 \cos(\frac{\pi}{3}n + \frac{\pi}{4})$



$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega})e^{-j\hat{\omega}}$$

## EX: COSINE INPUT (ans-1)

Find  $y[n]$  when  $x[n] = 2 \cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$2 \cos(\frac{\pi}{3}n + \frac{\pi}{4}) = e^{j(\pi n/3 + \pi/4)} + e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow x[n] = x_1[n] + x_2[n]$$

$$y_1[n] = H(e^{j\pi/3}) e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3}) e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow y[n] = y_1[n] + y_2[n]$$

### EX: COSINE INPUT (ans-2)

Find  $y[n]$  when  $x[n] = 2 \cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$H(e^{j\hat{\omega}}) = (2 + 2 \cos \hat{\omega})e^{-j\hat{\omega}}$$

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3 + \pi/4)} = 3e^{-j(\pi/3)}e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3 + \pi/4)} = 3e^{j(\pi/3)}e^{-j(\pi n/3 + \pi/4)}$$

$$y[n] = 3e^{j(\pi n/3 - \pi/12)} + 3e^{-j(\pi n/3 - \pi/12)}$$

$$\Rightarrow y[n] = 6 \cos(\frac{\pi}{3}n - \frac{\pi}{12})$$

19

### SINUSOID thru FIR

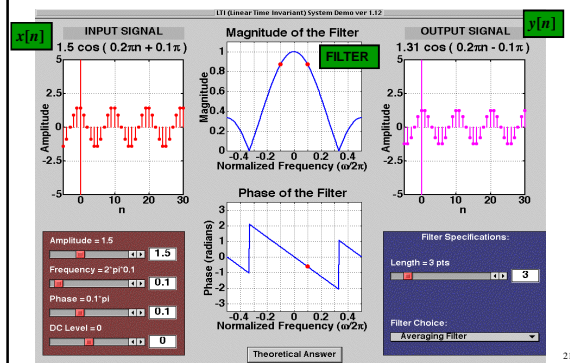
- IF  $H^*(e^{j\hat{\omega}}) = H(e^{-j\hat{\omega}})$
- Multiply the Magnitudes
- Add the Phases

$$x[n] = A \cos(\hat{\omega}_1 n + \phi)$$

$$\Rightarrow y[n] = A |H(e^{j\hat{\omega}_1})| \cos(\hat{\omega}_1 n + \phi + \angle H(e^{j\hat{\omega}_1}))$$

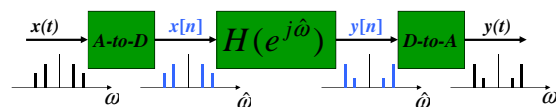
20

### LTI Demo with Sinusoids



21

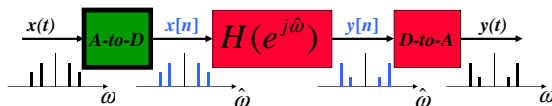
### DIGITAL "FILTERING"



- $\omega$  - SPECTRUM of  $x(t)$  (SUM of SINUSOIDS)
- $\hat{\omega}$  - SPECTRUM of  $x[n]$ 
  - Is ALIASING a PROBLEM ?
- $\hat{\omega}$  - SPECTRUM  $y[n]$  (FIR Gain or Nulls)
- $\omega$  - Then, OUTPUT  $y(t)$  = SUM of SINUSOIDS

22

### FREQUENCY SCALING

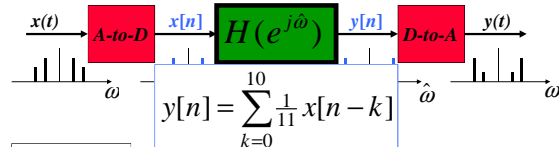


- TIME SAMPLING:  $t = nT_s$
- IF NO ALIASING:
- FREQUENCY SCALING

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

23

### 11-pt AVERAGER Example



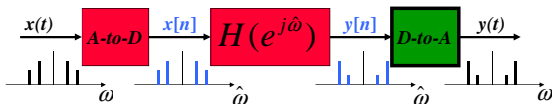
$$y[n] = \sum_{k=0}^{10} \frac{1}{11} x[n-k]$$

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{11 \sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

$$x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \frac{1}{2}\pi)$$

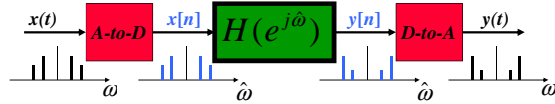
24

### D-A FREQUENCY SCALING



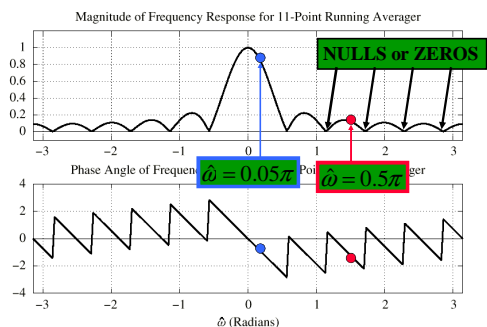
- TIME SAMPLING:  $t = nT_s \Rightarrow n \leftarrow tf_s$
- RECONSTRUCT up to  $0.5f_s$   
 - FREQUENCY SCALING  $\omega = \hat{\omega}f_s$

### TRACK the FREQUENCIES



- 250 Hz  $\rightarrow 0.5\pi$   $H(e^{j0.5\pi})$   $\rightarrow 0.5\pi$  • 250 Hz
  - 25 Hz  $\rightarrow .05\pi$   $H(e^{j0.05\pi})$   $\rightarrow .05\pi$  • 25 Hz
- $F_s = 1000$  Hz      NO new freqs

### 11-pt AVERAGER



### EVALUATE Freq. Response

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{11\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

At  $\hat{\omega} = 0.5\pi$

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}(0.5\pi))}{11\sin(\frac{1}{2}(0.5\pi))} e^{-j5(0.5\pi)}$$

$$= \frac{\sin(2.75\pi)}{11\sin(0.25\pi)} e^{-j2.5\pi}$$

$$= 0.0909 e^{-j0.5\pi}$$

### EVALUATE Freq. Response

$x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t)$   
 evaluating at 25 and 250 Hz.

$$H(e^{j2\pi(25)/1000}) = \frac{\sin(\pi(25)(11)/1000)}{11\sin(\pi(25)/1000)} e^{-j2\pi(25)(5)/1000}$$

$F_s = 1000$       MAG SCALE

$$= 0.8811 e^{-j\pi/4}$$

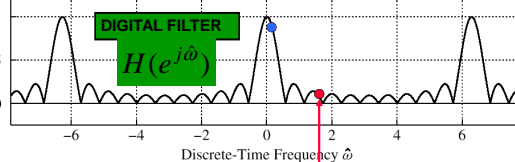
PHASE CHANGE

$$H(e^{j2\pi(250)/1000}) = \frac{\sin(\pi(250)(11)/1000)}{11\sin(\pi(250)/1000)} e^{-j2\pi(250)(5)/1000}$$

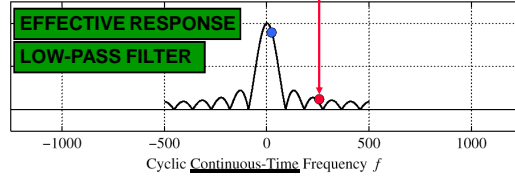
$$= 0.0909 e^{-j\pi/2}$$

$y(t) = 0.8811 \cos(2\pi(25)t - \pi/4) + 0.0909 \sin(2\pi(250)t - \pi/2)$

Magnitude of Frequency Response for 11-Point Running Averager

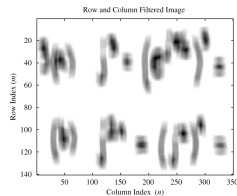


Equivalent Continuous-Time Frequency Response for  $f_s = 1000$



## FILTER TYPES

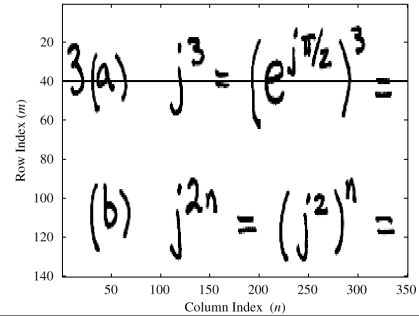
- LOW-PASS FILTER (**LPF**)
  - BLURRING
  - ATTENUATES HIGH FREQUENCIES
- HIGH-PASS FILTER (**HPF**)
  - SHARPENING for IMAGES
  - BOOSTS THE HIGHS
  - REMOVES DC
- BAND-PASS FILTER (**BPF**)



31

## B & W IMAGE

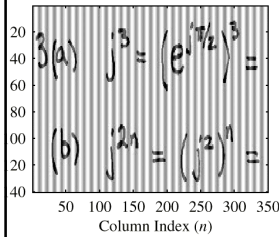
Original Black and White Image



32

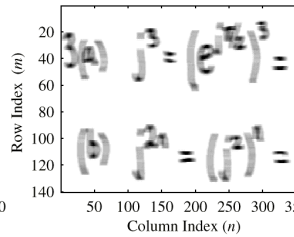
## B&W IMAGE with COSINE

Homework plus Cosine



**FILTERED: 11-pt AVG**

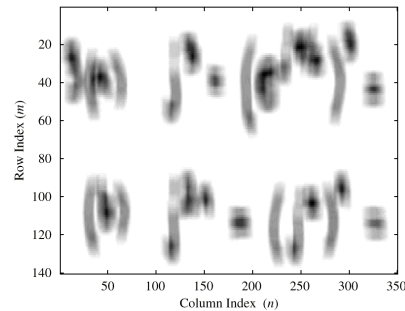
Remove Cosine Stripe with Averaging F



33

## FILTERED B&W IMAGE

Row and Column Filtered Image



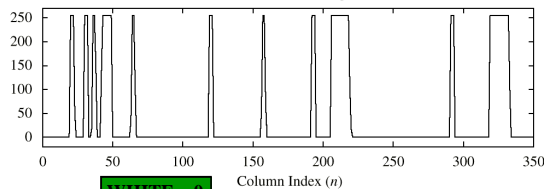
**LPF:  
BLUR**

34

## ROW of B&W IMAGE

**BLACK = 255**

Row 40 of the Image

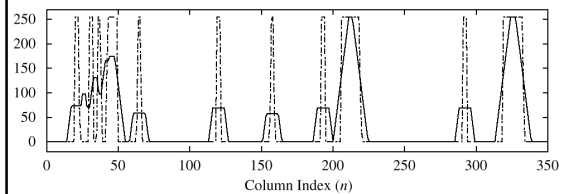


**WHITE = 0**

35

## FILTERED ROW of IMAGE

11-Point Averaging: 5-Sample Delay Equalization

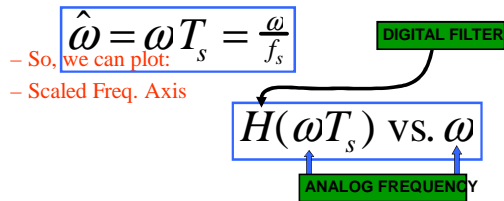


**ADJUSTED DELAY by 5 samples**

36

## EFFECTIVE Freq. Response

- Assume NO Aliasing, then
  - ANALOG FREQ  $\leftrightarrow$  DIGITAL FREQ



37

## TIME & FREQ DOMAINS

- LTI: Linear & Time-Invariant
  - COMPLETELY CHARACTERIZED by:
    - IMPULSE RESPONSE  $h[n]$  (time domain)
    - FREQUENCY RESPONSE



- Two DOMAINS: time & frequency
  - Go back and forth QUICKLY

38

## SINUSOID thru FIR

$$\begin{aligned}
 x[n] &= X_0 + \sum_{k=1}^N \left( \frac{X_k}{2} e^{j\hat{\omega}_k n} + \frac{X_k^*}{2} e^{-j\hat{\omega}_k n} \right) \\
 &= X_0 + \sum_{k=1}^N |X_k| \cos(\hat{\omega}_k n + \angle X_k)
 \end{aligned}$$

if  $\mathcal{H}(-\hat{\omega}) = \mathcal{H}^*(\hat{\omega})$ , the corresponding output is

$$\begin{aligned}
 y[n] &= \mathcal{H}(0)X_0 + \sum_{k=1}^N \left( \mathcal{H}(\hat{\omega}_k) \frac{X_k}{2} e^{j\hat{\omega}_k n} + \mathcal{H}(-\hat{\omega}_k) \frac{X_k^*}{2} e^{-j\hat{\omega}_k n} \right) \\
 &= \mathcal{H}(0)X_0 + \sum_{k=1}^N \boxed{|\mathcal{H}(\hat{\omega}_k)|} |X_k| \cos(\hat{\omega}_k n + \angle X_k + \boxed{\angle \mathcal{H}(\hat{\omega}_k)})
 \end{aligned}$$

Annotations:
 

- 'MULTIPLY MAGS' points to the magnitude terms in the final equation.
- 'ADD PHASES' points to the phase terms in the final equation.

39