Digital Signal Processing

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Digital Signal Processing

Lecture 13

Digital Filtering of Analog Signals

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READING ASSIGNMENTS

- This Lecture:
  - Chapter 6, Sections 6-6, 6-7 & 6-8
- Other Reading:
  - Recitation: Chapter 6
    - FREQUENCY RESPONSE EXAMPLES
  - Next Lecture: Chapter 7

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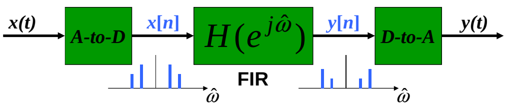
LECTURE OBJECTIVES

- Two Domains: Time & Frequency
- Track the spectrum of x[n] thru an FIR Filter: **Sinusoid-IN gives Sinusoid-OUT**
- UNIFICATION: How does Frequency Response affect $x(t)$ to produce $y(t)$?



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TIME & FREQUENCY

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

FIR DIFFERENCE EQUATION is the TIME-DOMAIN

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} h[k]e^{-j\hat{\omega}k}$$

$$H(e^{j\hat{\omega}}) = h[0] + h[1]e^{-j\hat{\omega}} + h[2]e^{-j2\hat{\omega}} + h[3]e^{-j3\hat{\omega}} + \cdots$$

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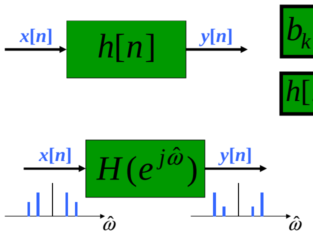
Ex: DELAY by 2 SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-2]$

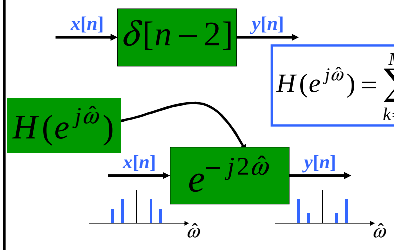


$b_k = \{0, 0, 1\}$

$h[n] = \delta[n-2]$

DELAY by 2 SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-2]$



$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} \delta[k-2] e^{-j\hat{\omega}k}$$

k = 2 ONLY

GENERAL DELAY PROPERTY

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-n_d]$

$h[n] = \delta[n-n_d]$

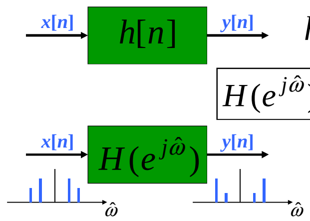
$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} \delta[k-n_d] e^{-j\hat{\omega}k} = e^{-j\hat{\omega}n_d}$$

ONLY ONE non-ZERO TERM for k at $k = n_d$

FREQ DOMAIN --> TIME ??

- START with

$H(e^{j\hat{\omega}})$ and find $h[n]$ or b_k



$h[n] = ?$

$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega})$

FREQ DOMAIN --> TIME

$H(e^{j\hat{\omega}}) = 7e^{-j2\hat{\omega}} \cos(\hat{\omega})$ **EULER's Formula**

$= 7e^{-j2\hat{\omega}}(0.5e^{j\hat{\omega}} + 0.5e^{-j\hat{\omega}})$

$= (3.5e^{-j\hat{\omega}} + 3.5e^{-j3\hat{\omega}})$

$h[n] = 3.5\delta[n-1] + 3.5\delta[n-3]$

$b_k = \{0, 3.5, 0, 3.5\}$

PREVIOUS LECTURE REVIEW

- SINUSOIDAL INPUT SIGNAL
  - OUTPUT has SAME FREQUENCY
  - DIFFERENT Amplitude and Phase
- FREQUENCY RESPONSE of FIR
  - MAGNITUDE vs. Frequency
  - PHASE vs. Freq
  - PLOTTING

MAG **PHASE**

$$H(e^{j\hat{\omega}}) = |H(e^{j\hat{\omega}})| e^{j\angle H(e^{j\hat{\omega}})}$$

FREQ. RESPONSE PLOTS

- DENSE GRID `(ww)` from $-\pi$ to $+\pi$
  - `ww = -pi:(pi/100):pi;`
- `HH = freqz(bb,1,ww)`
  - VECTOR `bb` contains Filter Coefficients
  - DSP-First: `HH = freekz(bb,1,ww)`

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

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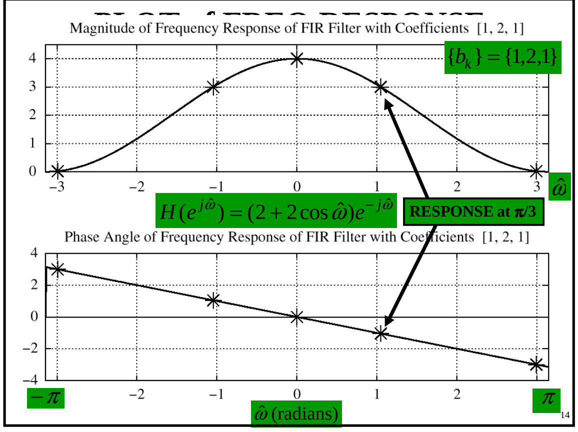

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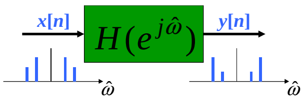
EXAMPLE 6.2

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known and $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$



$$H(e^{j\hat{\omega}}) = (2+2\cos\hat{\omega})e^{-j\hat{\omega}}$$

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EXAMPLE 6.2 (answer)

Find $y[n]$ when $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$

One Step - evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/3$

$$H(e^{j\hat{\omega}}) = (2+2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$H(e^{j\hat{\omega}}) = 3e^{-j\pi/3} \quad @\, \hat{\omega} = \pi/3$$

$$y[n] = \left(3e^{-j\pi/3}\right) \times 2e^{j\pi/4}e^{j(\pi/3)n} = 6e^{-j\pi/12}e^{j(\pi/3)n}$$

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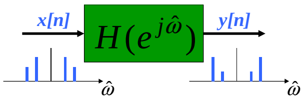
EXAMPLE: COSINE INPUT

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known and $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$



$$H(e^{j\hat{\omega}}) = (2+2\cos\hat{\omega})e^{-j\hat{\omega}}$$

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EX: COSINE INPUT (ans-1)

Find $y[n]$ when $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$2\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4}) = e^{j(\pi n/3 + \pi/4)} + e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow x[n] = x_1[n] + x_2[n]$$

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow y[n] = y_1[n] + y_2[n]$$

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EX: COSINE INPUT (ans-2)

Find $y[n]$ when $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$H(e^{j\hat{\omega}}) = (2 + 2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3+\pi/4)} = 3e^{-j(\pi/3)}e^{j(\pi n/3+\pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3+\pi/4)} = 3e^{j(\pi/3)}e^{-j(\pi n/3+\pi/4)}$$

$$y[n] = 3e^{j(\pi n/3-\pi/12)} + 3e^{-j(\pi n/3-\pi/12)}$$

$$\Rightarrow y[n] = 6\cos(\tfrac{\pi}{3}n - \tfrac{\pi}{12})$$

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SINUSOID thru FIR

- IF $H^*(e^{j\hat{\omega}}) = H(e^{-j\hat{\omega}})$
- Multiply the Magnitudes
- Add the Phases

$$x[n] = A\cos(\hat{\omega}_1 n + \phi)$$

$$\Rightarrow y[n] = A\left|H(e^{j\hat{\omega}_1})\right|\cos(\hat{\omega}_1 n + \phi + \angle H(e^{j\hat{\omega}_1}))$$

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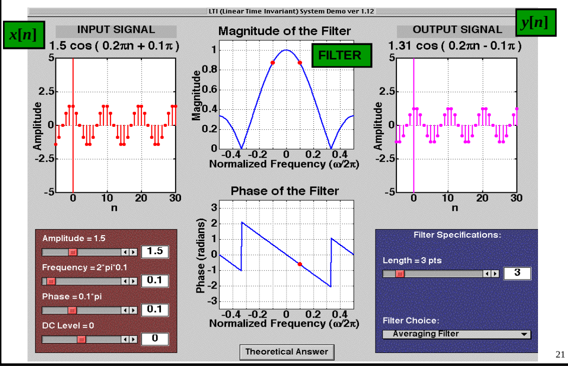
LTI Demo with Sinusoids



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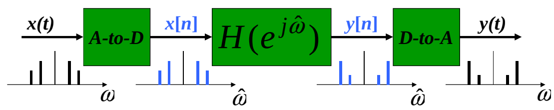
DIGITAL "FILTERING"



- ω – SPECTRUM of x(t) (SUM of SINUSOIDS)
- $\hat{\omega}$ – SPECTRUM of x[n]
  - Is ALIASING a PROBLEM ?
- – SPECTRUM y[n] (FIR Gain or Nulls)
- ω – Then, OUTPUT y(t) = SUM of SINUSOIDS

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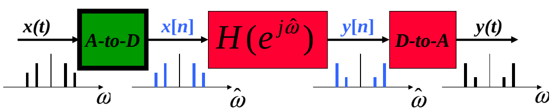
FREQUENCY SCALING



- TIME SAMPLING: $t = nT_s$
  - IF NO ALIASING:
  - FREQUENCY SCALING

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

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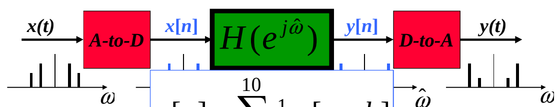
11-pt AVERAGER Example



$$y[n] = \sum_{k=0}^{10} \tfrac{1}{11} x[n-k]$$

250 Hz

25 Hz

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{11\sin(\frac{1}{2}\hat{\omega})}e^{-j5\hat{\omega}}$$?

$$x(t) = \cos(2\pi(25)t) + \cos(2\pi(250)t - \tfrac{1}{2}\pi)$$

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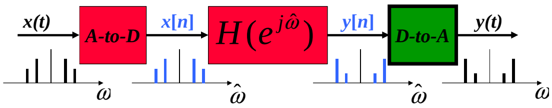
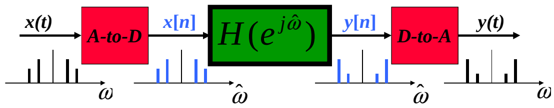
D-A FREQUENCY SCALING



- TIME SAMPLING: $t = nT_s \Rightarrow n \leftarrow t f_s$
- RECONSTRUCT up to $0.5f_s$
  - FREQUENCY SCALING

$$\omega = \hat{\omega} f_s$$

TRACK the FREQUENCIES

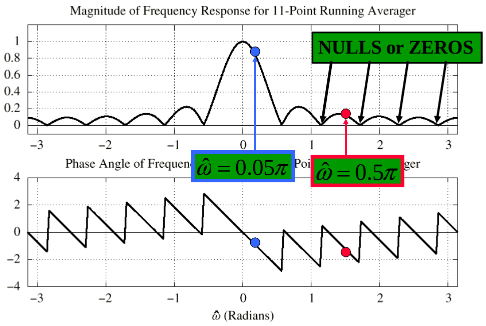


• 250 Hz	• 0.5π	$H(e^{j0.5\pi})$	• 0.5π	• 250 Hz
• 25 Hz	• .05π	$H(e^{j0.05\pi})$	• .05π	• 25 Hz

Fs = 1000 Hz

NO new freqs

11-pt AVERAGER



EVALUATE Freq. Response

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}\hat{\omega})}{11\sin(\frac{1}{2}\hat{\omega})} e^{-j5\hat{\omega}}$$

At $\hat{\omega} = 0.5\pi$

$$H(e^{j\hat{\omega}}) = \frac{\sin(\frac{11}{2}(0.5\pi))}{11\sin(\frac{1}{2}(0.5\pi))} e^{-j5(0.5\pi)}$$

$$= \frac{\sin(2.75\pi)}{11\sin(0.25\pi)} e^{-j2.5\pi}$$

$$= 0.0909 e^{-j0.5\pi}$$

EVALUATE Freq. Response

$$x(t) = \cos(2\pi(25)t) + \sin(2\pi(250)t)$$

evaluating at 25 and 250 Hz.

$$H(e^{j2\pi(25)/1000}) = \frac{\sin(\pi(25)(11)/1000)}{11\sin(\pi(25)/1000)} e^{-j2\pi(25)(5)/1000}$$

$f_s = 1000$

MAG SCALE

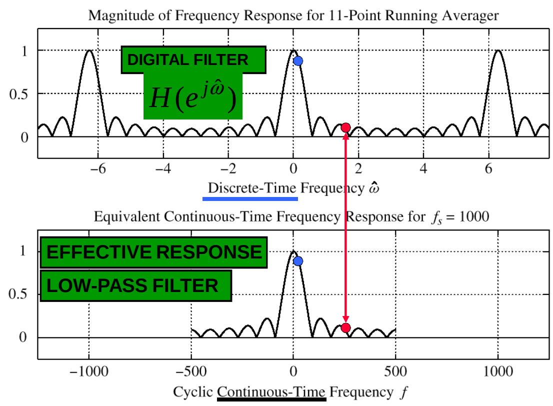
$$= 0.8811 e^{-j\pi/4}$$

PHASE CHANGE

$$H(e^{j2\pi(250)/1000}) = \frac{\sin(\pi(250)(11)/1000)}{11\sin(\pi(250)/1000)} e^{-j2\pi(250)(5)/1000}$$

$$= 0.0909 e^{-j\pi/2}$$

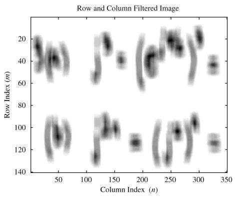
$$y(t) = 0.8811\cos(2\pi(25)t - \pi/4) + 0.0909\sin(2\pi(250)t - \pi/2)$$



FILTER TYPES

- LOW-PASS FILTER (LPF)
  - BLURRING
  - ATTENUATES HIGH FREQUENCIES
- HIGH-PASS FILTER (HPF)
  - SHARPENING for IMAGES
  - BOOSTS THE HIGHS
  - REMOVES DC
- BAND-PASS FILTER

  (BPF)



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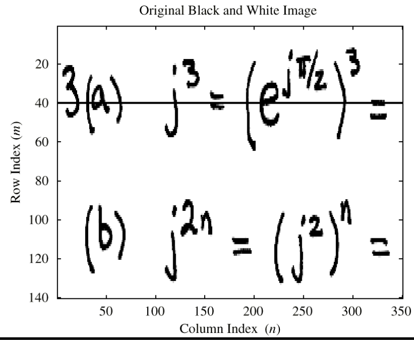
B & W IMAGE



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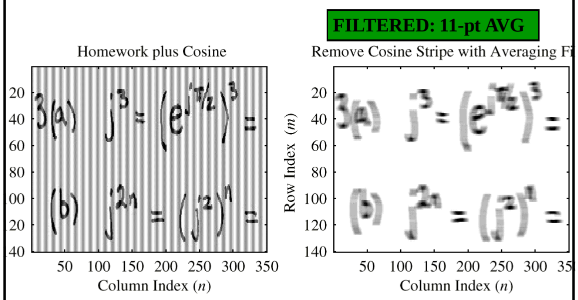
B&W IMAGE with COSINE



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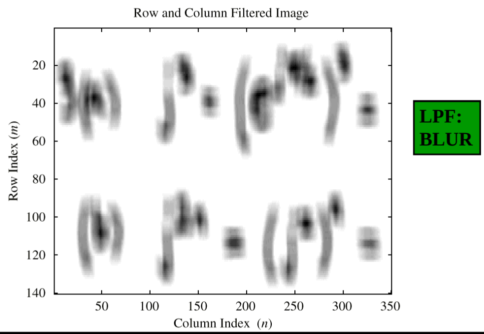
FILTERED B&W IMAGE



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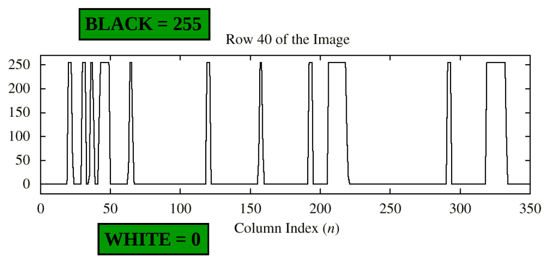
ROW of B&W IMAGE



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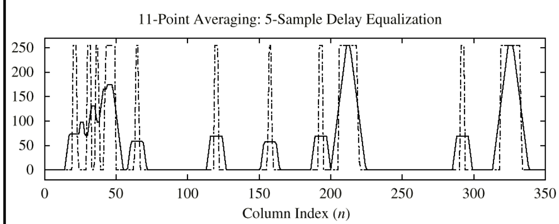
FILTERED ROW of IMAGE



ADJUSTED DELAY by 5 samples

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EFFECTIVE Freq. Response

- Assume NO Aliasing, then
  - ANALOG FREQ <--> DIGITAL FREQ

$$\hat{\omega} = \omega T_s = \frac{\omega}{f_s}$$

- So, we can plot:
- Scaled Freq. Axis

DIGITAL FILTER

$$H(\omega T_s) \text{ vs. } \omega$$

ANALOG FREQUENCY

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TIME & FREQ DOMAINS

- LTI: Linear & Time-Invariant
  - COMPLETELY CHARACTERIZED by:
    - IMPULSE RESPONSE h[n] (time domain)
    - FREQUENCY RESPONSE

x[n] → $H(\hat{\omega}) = |H(\hat{\omega})| e^{j\varphi(\hat{\omega})}$ → y[n]

- Two DOMAINS: time & frequency
  - Go back and forth QUICKLY

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SINUSOID thru FIR

$$x[n] = X_0 + \sum_{k=1}^{N} \left(\frac{X_k}{2} e^{j\hat{\omega}_k n} + \frac{X_k^*}{2} e^{-j\hat{\omega}_k n} \right)$$

$$= X_0 + \sum_{k=1}^{N} |X_k| \cos(\hat{\omega}_k n + \angle X_k)$$

MULTIPLY MAGS

ADD PHASES

if $\mathcal{H}(-\hat{\omega}) = \mathcal{H}^*(\hat{\omega})$, the corresponding output is

$$y[n] = \mathcal{H}(0) X_0 + \sum_{k=1}^{N} \left(\mathcal{H}(\hat{\omega}_k) \frac{X_k}{2} e^{j\hat{\omega}_k n} + \mathcal{H}(-\hat{\omega}_k) \frac{X_k^*}{2} e^{-j\hat{\omega}_k n} \right)$$

$$= \mathcal{H}(0) X_0 + \sum_{k=1}^{N} |\mathcal{H}(\hat{\omega}_k)| |X_k| \cos\left(\hat{\omega}_k n + \angle X_k + \angle \mathcal{H}(\hat{\omega}_k)\right)$$

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