# Digital Signal Processing

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# Digital Signal Processing

Lecture 12

## Frequency Response of FIR Filters



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## READING ASSIGNMENTS

- This Lecture:
  - Chapter 6, Sections 6-1, 6-2, 6-3, 6-4, & 6-5
- Other Reading:
  - Recitation: Chapter 6
    - FREQUENCY RESPONSE EXAMPLES
  - Next Lecture: Chap. 6, Sects. 6-6, 6-7 & 6-8



## LECTURE OBJECTIVES

- SINUSOIDAL INPUT SIGNAL
  - DETERMINE the FIR FILTER OUTPUT
- FREQUENCY RESPONSE of FIR
  - PLOTTING vs. Frequency
  - MAGNITUDE vs. Freq
  - PHASE vs. Freq

MAG PHASE

$$H(e^{j\hat{\omega}}) = |H(e^{j\hat{\omega}})| e^{j\angle H(e^{j\hat{\omega}})}$$



## DOMAINS: Time & Frequency

- **Time-Domain: "$n$" = time**
  - $x[n]$ discrete-time signal
  - $x(t)$ continuous-time signal
- **Frequency Domain (sum of sinusoids)**
  - Spectrum vs. $f$ (Hz)
    - ANALOG vs. DIGITAL
  - Spectrum vs. omega-hat
- Move back and forth QUICKLY

## LTI SYSTEMS

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - **IMPULSE RESPONSE** $h[n]$
  - **CONVOLUTION**: $y[n] = x[n]*h[n]$
    - The "rule" defining the system can ALWAYS be re-written as convolution
- FIR Example: $h[n]$ is same as $b_k$

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## POP QUIZ

- FIR Filter is "FIRST DIFFERENCE"
  - $y[n] = x[n] - x[n-1]$
- Write output as a convolution
  - Need impulse response

$$h[n] = \delta[n] - \delta[n-1]$$

  - Then, another way to compute the output:

$$y[n] = (\delta[n] - \delta[n-1]) * x[n]$$

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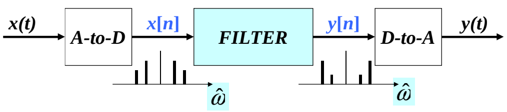
## DIGITAL "FILTERING"




- CONCENTRATE on the **SPECTRUM**
- SINUSOIDAL INPUT
  - INPUT $x[n]$ = SUM of SINUSOIDS
  - Then, OUTPUT $y[n]$ = SUM of SINUSOIDS

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## FILTERING EXAMPLE

- 7-point AVERAGER
  - Removes cosine
    - By making its amplitude (A) smaller

$$y_7[n] = \sum_{k=0}^{6} \left(\tfrac{1}{7}\right) x[n-k]$$

- 3-point AVERAGER
  - Changes A slightly

$$y_3[n] = \sum_{k=0}^{2} \left(\tfrac{1}{3}\right) x[n-k]$$

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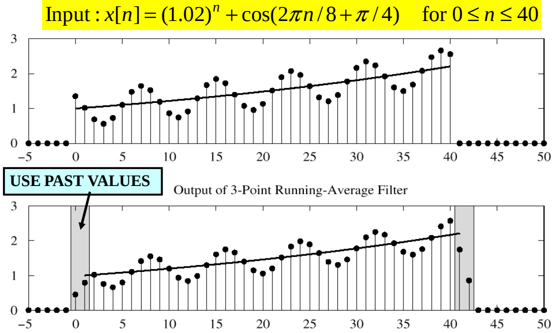
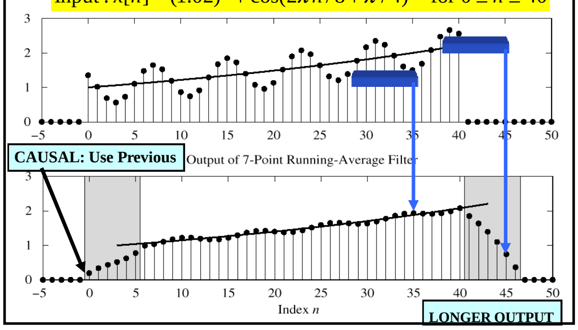
## 3-pt AVG EXAMPLE

Input : $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$




## 7-pt FIR EXAMPLE (AVG)

Input : $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4) \quad \text{for } 0 \le n \le 40$

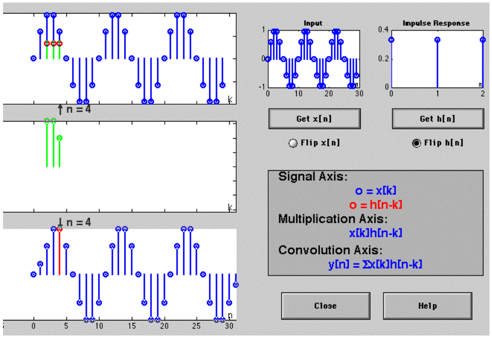


## SINUSOIDAL RESPONSE

- INPUT: x[n] = SINUSOID
- OUTPUT: y[n] will also be a SINUSOID
  - Different Amplitude and Phase
  - **SAME** Frequency
- AMPLITUDE & PHASE CHANGE
  - Called the **FREQUENCY RESPONSE**



## DCONVDEMO: MATLAB GUI






## COMPLEX EXPONENTIAL

$$x[n] = Ae^{j\varphi}e^{j\hat{\omega}n} \qquad -\infty < n < \infty$$

**x[n] is the input signal—a complex exponential**

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} h[k]x[n-k]$$

**FIR DIFFERENCE EQUATION**



## COMPLEX EXP OUTPUT

- Use the FIR "Difference Equation"

$$y[n] = \sum_{k=0}^{M} b_k x[n-k] = \sum_{k=0}^{M} b_k Ae^{j\varphi}e^{j\hat{\omega}(n-k)}$$

$$= \left(\sum_{k=0}^{M} b_k e^{j\hat{\omega}(-k)}\right) Ae^{j\varphi}e^{j\hat{\omega}n}$$

$$= H(\hat{\omega})Ae^{j\varphi}e^{j\hat{\omega}n}$$



## FREQUENCY RESPONSE

- At each frequency, we can DEFINE

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$ ← **FREQUENCY RESPONSE**

- Complex-valued formula
  - Has MAGNITUDE vs. frequency
  - And PHASE vs. frequency
- Notation: $H(e^{j\hat{\omega}})$ in place of $H(\hat{\omega})$



## EXAMPLE 6.1

$$\{b_k\} = \{1,\ 2,\ 1\}$$

$$H(e^{j\hat{\omega}}) = 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

$$= e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}})$$

$$= e^{-j\hat{\omega}}(2 + 2\cos\hat{\omega})$$

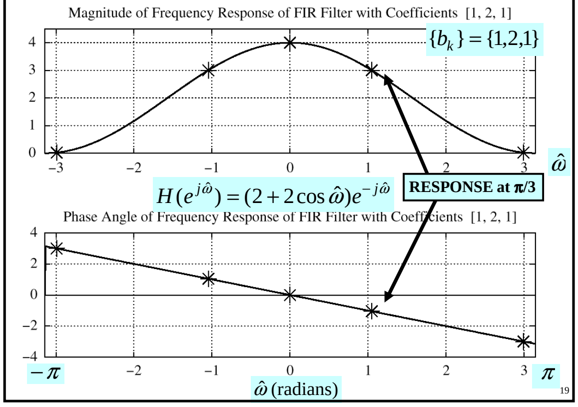
**EXPLOIT SYMMETRY**

Since $(2 + 2\cos\hat{\omega}) \geq 0$

Magnitude is $|H(e^{j\hat{\omega}})| = (2 + 2\cos\hat{\omega})$

and Phase is $\angle H(e^{j\hat{\omega}}) = -\hat{\omega}$

## PLOT of FREQ RESPONSE

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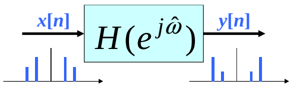
## EXAMPLE 6.2

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known and $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$




$$H(e^{j\hat{\omega}}) = (2+2\cos\hat{\omega})e^{-j\hat{\omega}}$$

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## EXAMPLE 6.2 (answer)

Find $y[n]$ when $x[n] = 2e^{j\pi/4}e^{j(\pi/3)n}$

One Step - evaluate $H(e^{j\hat{\omega}})$ at $\hat{\omega} = \pi/3$

$$H(e^{j\hat{\omega}}) = (2+2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$H(e^{j\hat{\omega}}) = 3e^{-j\pi/3} \quad @\,\hat{\omega} = \pi/3$$

$$y[n] = \left(3e^{-j\pi/3}\right) \times 2e^{j\pi/4}e^{j(\pi/3)n} = 6e^{-j\pi/12}e^{j(\pi/3)n}$$

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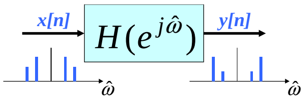
## EXAMPLE: COSINE INPUT

Find $y[n]$ when $H(e^{j\hat{\omega}})$ is known and $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$




$$H(e^{j\hat{\omega}}) = (2+2\cos\hat{\omega})e^{-j\hat{\omega}}$$

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## EX: COSINE INPUT

Find $y[n]$ when $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$2\cos(\tfrac{\pi}{3}n + \tfrac{\pi}{4}) = e^{j(\pi n/3 + \pi/4)} + e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow x[n] = x_1[n] + x_2[n]$$

Use Linearity

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3 + \pi/4)}$$

$$\Rightarrow y[n] = y_1[n] + y_2[n]$$

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## EX: COSINE INPUT (ans-2)

Find $y[n]$ when $x[n] = 2\cos(\frac{\pi}{3}n + \frac{\pi}{4})$

$$H(e^{j\hat{\omega}}) = (2+2\cos\hat{\omega})e^{-j\hat{\omega}}$$

$$y_1[n] = H(e^{j\pi/3})e^{j(\pi n/3 + \pi/4)} = 3e^{-j(\pi/3)}e^{j(\pi n/3 + \pi/4)}$$

$$y_2[n] = H(e^{-j\pi/3})e^{-j(\pi n/3 + \pi/4)} = 3e^{j(\pi/3)}e^{-j(\pi n/3 + \pi/4)}$$

$$y[n] = 3e^{j(\pi n/3 - \pi/12)} + 3e^{-j(\pi n/3 - \pi/12)}$$

$$\Rightarrow y[n] = 6\cos(\tfrac{\pi}{3}n - \tfrac{\pi}{12})$$

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## MATLAB: FREQUENCY RESPONSE

- `HH = freqz(bb,1,ww)`
  - VECTOR `bb` contains Filter Coefficients
  - SP-First: `HH = freekz(bb,1,ww)`
- FILTER COEFFICIENTS $\{b_k\}$

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k}$$

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## LTI SYSTEMS

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - <u>FREQUENCY RESPONSE</u>, or
  - IMPULSE RESPONSE $h[n]$
- <u>Sinusoid IN -----> Sinusoid OUT</u>
  - <u>At the SAME Frequency</u>

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## Time & Frequency Relation

- Get Frequency Response from $h[n]$
  - Here is the FIR case:

$$H(e^{j\hat{\omega}}) = \sum_{k=0}^{M} b_k e^{-j\hat{\omega}k} = \sum_{k=0}^{M} h[k] e^{-j\hat{\omega}k}$$

IMPULSE RESPONSE

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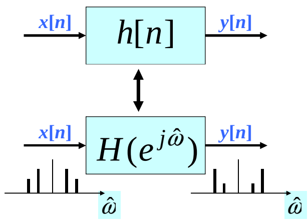
## BLOCK DIAGRAMS

- Equivalent Representations

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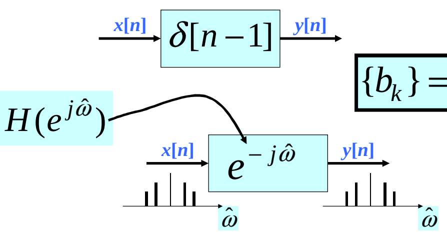
## UNIT-DELAY SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for $y[n] = x[n-1]$




$$\{b_k\} = \{\,0,\ 1\,\}$$

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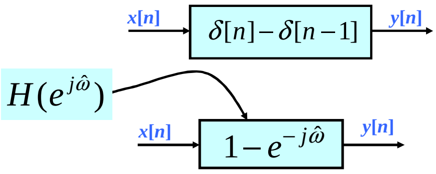
## FIRST DIFFERENCE SYSTEM

Find $h[n]$ and $H(e^{j\hat{\omega}})$ for the Difference Equation: $y[n] = x[n] - x[n-1]$

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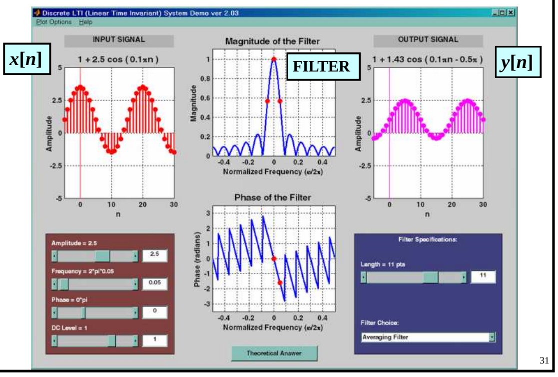
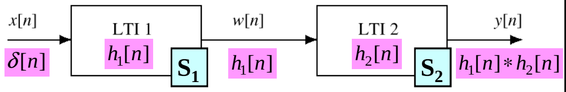
# DLTI Demo with Sinusoids






# CASCADE SYSTEMS

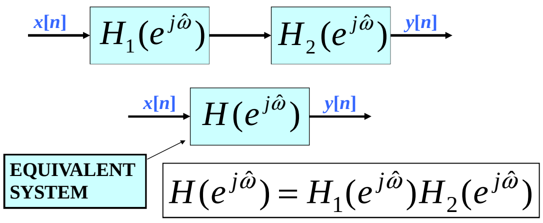
- Does the order of $S_1$ & $S_2$ matter?
  - NO, LTI SYSTEMS can be rearranged !!!
  - WHAT ARE THE FILTER COEFFS? $\{b_k\}$
  - WHAT is the overall FREQUENCY RESPONSE ?






# CASCADE EQUIVALENT

- MULTIPLY the Frequency Responses




$$H(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}})$$