Digital Signal Processing

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Digital Signal Processing

Lecture 11

Linearity & Time-Invariance
Convolution

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License Info for SPFirst Slides

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READING ASSIGNMENTS

- This Lecture:
  - Chapter 5, Sections 5-5 and 5-6
    - Section 5-4 will be covered, but not "in depth"
- Other Reading:
  - Recitation: Ch. 5, Sects 5-6, 5-7 & 5-8
    - CONVOLUTION
  - Next Lecture: start Chapter 6

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LECTURE OBJECTIVES

- GENERAL PROPERTIES of FILTERS
  - LINEARITY
  - TIME-INVARIANCE
  - ==> **CONVOLUTION**

  LTI SYSTEMS
  - BLOCK DIAGRAM REPRESENTATION
    - Components for Hardware
    - Connect Simple Filters Together to Build More Complicated Systems

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OVERVIEW

- IMPULSE RESPONSE, $h[n]$
  - FIR case: same as $\{b_k\}$
- CONVOLUTION
  - GENERAL: $y[n] = h[n] * x[n]$
  - GENERAL CLASS of SYSTEMS
  - LINEAR and TIME-INVARIANT
- ALL **LTI** systems have $\boldsymbol{h[n]}$ & use convolution

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DIGITAL FILTERING

- CONCENTRATE on the FILTER (DSP)
- DISCRETE-TIME SIGNALS
  - FUNCTIONS of ***n***, the "time index"
  - INPUT ***x*[*n*]**
  - OUTPUT ***y*[*n*]**

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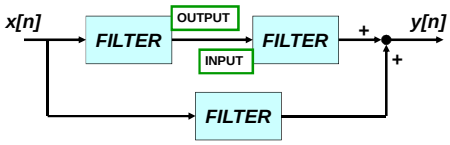
BUILDING BLOCKS



- BUILD UP COMPLICATED FILTERS
  - FROM SIMPLE MODULES
  - Ex: FILTER MODULE MIGHT BE 3-pt FIR

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GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

  - For example, $b_k = \{3, -1, 2, 1\}$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$
$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

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MATLAB for FIR FILTER

- `yy = conv(bb,xx)`
  - VECTOR **bb** contains Filter Coefficients
  - DSP-First: `yy = firfilt(bb,xx)`
- FILTER COEFFICIENTS $\{b_k\}$

conv2() *for images*

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

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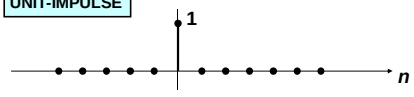
SPECIAL INPUT SIGNALS

- $x[n]$ = SINUSOID **FREQUENCY RESPONSE**
- $x[n]$ has only one NON-ZERO VALUE

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

UNIT-IMPULSE



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FIR IMPULSE RESPONSE

- Convolution = Filter Definition
  - Filter Coeffs = Impulse Response

n	$n<0$	0	1	2	3	...	M	$M+1$	$n>M+1$
$x[n]=\delta[n]$	0	1	0	0	0	0	0	0	0
$y[n]=h[n]$	0	b_0	b_1	b_2	b_3	...	b_M	0	0

$$h[n] = \sum_{k=0}^{M} b_k \delta[n-k]$$

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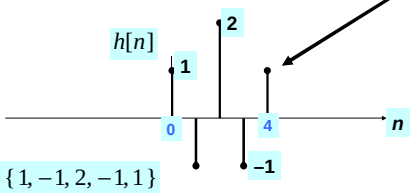
MATH FORMULA for *h*[*n*]

- Use SHIFTED IMPULSES to write ***h***[***n***]

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$



$$b_k = \{1, -1, 2, -1, 1\}$$

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LTI: Convolution Sum

- **Output = Convolution of *x*[*n*] & *h*[*n*]**
  - NOTATION: $y[n] = h[n] * x[n]$
  - Here is the FIR case:

FINITE LIMITS

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

Same as b_k FINITE LIMITS

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CONVOLUTION Example

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$$
$$x[n] = u[n]$$

n	–1	0	1	2	3	4	5	6	7
$x[n]$	0	1	1	1	1	1	1	1	...
$h[n]$	0	1	–1	2	–1	1	0	0	0
$h[0]x[n]$	0	1	1	1	1	1	1	1	1
$h[1]x[n-1]$	0	0	–1	–1	–1	–1	–1	–1	–1
$h[2]x[n-2]$	0	0	0	2	2	2	2	2	2
$h[3]x[n-3]$	0	0	0	0	–1	–1	–1	–1	–1
$h[4]x[n-4]$	0	0	0	0	0	1	1	1	1
$y[n]$	0	1	0	2	1	2	2	2	...

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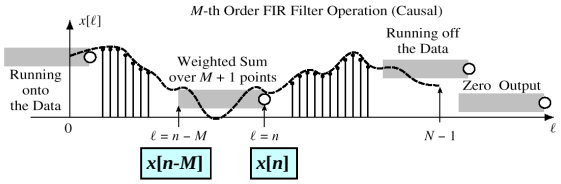
GENERAL FIR FILTER

- SLIDE a Length-L WINDOW over x[n]

$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k]$$



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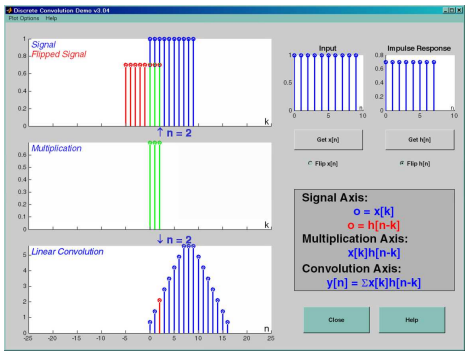
DCONVDEMO: MATLAB GUI



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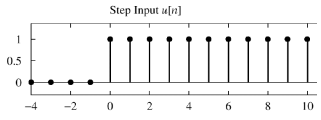
POP QUIZ

- FIR Filter is "FIRST DIFFERENCE"
  $y[n] = x[n] - x[n-1]$
- INPUT is "UNIT STEP"

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



- Find ***y***[***n***]

$$y[n] = u[n] - u[n-1] = \delta[n]$$

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HARDWARE STRUCTURES

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

- INTERNAL STRUCTURE of "FILTER"
  - WHAT COMPONENTS ARE NEEDED?
  - HOW DO WE "HOOK" THEM TOGETHER?
- SIGNAL FLOW GRAPH NOTATION

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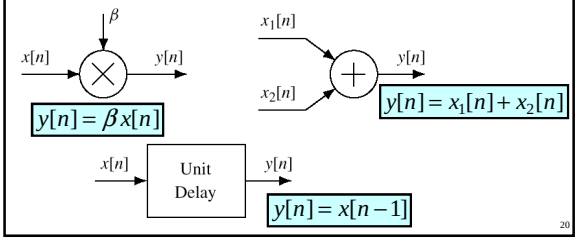
HARDWARE ATOMS

- Add, Multiply & Store

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$



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FIR STRUCTURE

- Direct Form

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

SIGNAL FLOW GRAPH

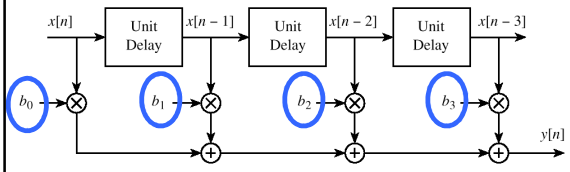
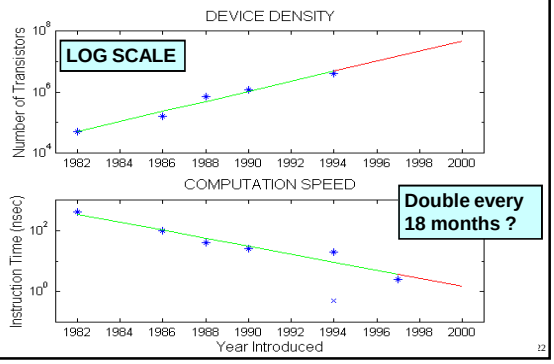


Figure 5.13 Block-diagram structure for the Mth order FIR filter.

Moore's Law for TI DSPs



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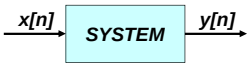
SYSTEM PROPERTIES



- MATHEMATICAL DESCRIPTION
- TIME-INVARIANCE
- LINEARITY
- CAUSALITY
  - "No output prior to input"

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TIME-INVARIANCE

- IDEA:
  - "Time-Shifting the input will cause the same time-shift in the output"
- EQUIVALENTLY,
  - We can prove that
    - The time origin (n=0) is picked arbitrary

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TESTING Time-Invariance

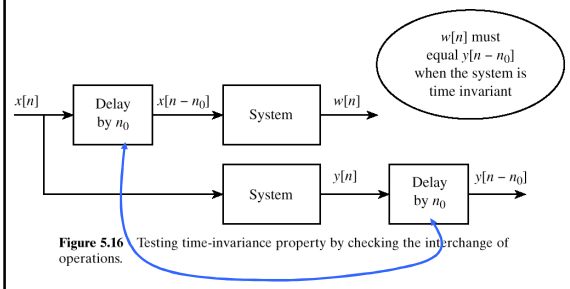

Figure 5.16 Testing time-invariance property by checking the interchange of operations.

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LINEAR SYSTEM

- LINEARITY = Two Properties
- SCALING
  - "Doubling $x[n]$ will double $y[n]$"
- SUPERPOSITION:
  - "Adding two inputs gives an output that is the sum of the individual outputs"

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TESTING LINEARITY

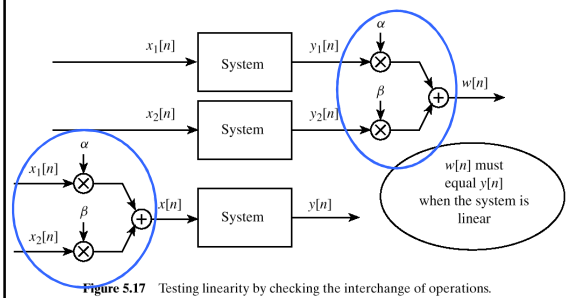

Figure 5.17 Testing linearity by checking the interchange of operations.

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LTI SYSTEMS

- LTI: Linear & Time-Invariant
- COMPLETELY CHARACTERIZED by:
  - **IMPULSE RESPONSE** $h[n]$
  - **CONVOLUTION**: $y[n] = x[n]*h[n]$
    - The "rule" defining the system can ALWAYS be re-written as convolution
- FIR Example: $h[n]$ is same as b_k

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POP QUIZ

- FIR Filter is "FIRST DIFFERENCE"
  - $y[n] = x[n] - x[n-1]$
- Write output as a convolution
  - Need impulse response

$$h[n] = \delta[n] - \delta[n-1]$$

  - Then, another way to compute the output:

$$y[n] = (\delta[n] - \delta[n-1]) * x[n]$$

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CASCADE SYSTEMS

- Does the order of S_1 & S_2 matter?
  - NO, LTI SYSTEMS can be rearranged !!!
  - WHAT ARE THE FILTER COEFFS? $\{b_k\}$

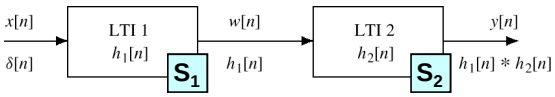

Figure 5.19 A Cascade of Two LTI Systems.

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CASCADE EQUIVALENT

– Find "overall" $h[n]$ for a cascade ?

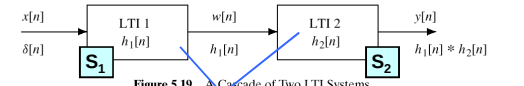

Figure 5.19 A Cascade of Two LTI Systems.

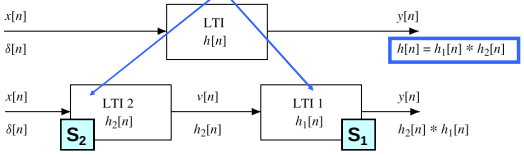

Figure 5.20 Switching the order of cascaded LTI systems.

CONVOLUTION Example

n	0	1	2	3	4	5	6	7	8
x[n]	2	4	6	4	2				
h[n]	3	-1	2	1					
h[0]x[n-0]	6	12	18	12	6				
h[1]x[n-1]		-2	-4	-6	-4	-2			
h[2]x[n-2]			4	8	12	8	4		
h[3]x[n-3]				2	4	6	4	2	
y[n]	6	10	18	16	18	12	8	2	