# Digital Signal Processing

Prof. Nizamettin AYDIN

naydin@yildiz.edu.tr

http://www.yildiz.edu.tr/~naydin

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# Digital Signal Processing

Lecture 10

## FIR Filtering

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## License Info for SPFirst Slides



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## READING ASSIGNMENTS

- This Lecture:
  - Chapter 5, Sects. 5-1, 5-2 and 5-3 (partial)
- Other Reading:
  - Recitation: Ch. 5, Sects 5-4, 5-6, 5-7 and 5-8
    - CONVOLUTION
  - Next Lecture: Ch 5, Sects. 5-3, 5-5 and 5-6

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## LECTURE OBJECTIVES

- INTRODUCE FILTERING IDEA
  - Weighted Average
  - Running Average
- FINITE IMPULSE RESPONSE FILTERS
  - **FIR** Filters
  - Show how to **compute** the output $y[n]$ from the input signal, $x[n]$

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## DIGITAL FILTERING


- CONCENTRATE on the COMPUTER
  - PROCESSING ALGORITHMS
  - SOFTWARE (MATLAB)
  - HARDWARE: DSP chips, VLSI
- DSP: DIGITAL SIGNAL PROCESSING

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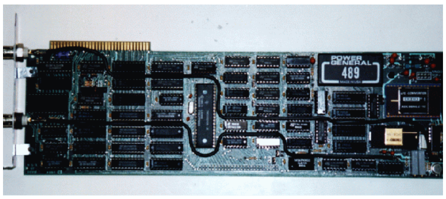
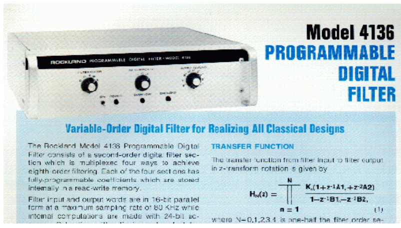
## The TMS32010, 1983



First PC plug-in board from Atlanta Signal Processors Inc.



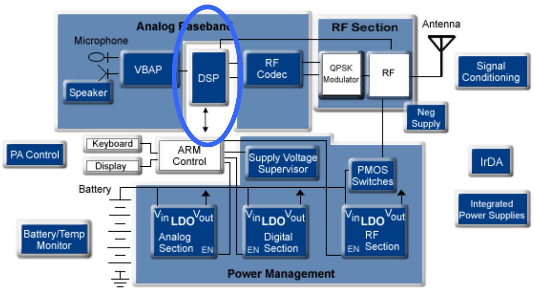
## Rockland Digital Filter, 1971




For the price of a small house, you could have one of these.



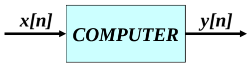
## Digital Cell Phone (ca. 2000)




Now it plays video



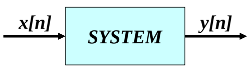
## DISCRETE-TIME SYSTEM




- OPERATE on $x[n]$ to get $y[n]$
- WANT a **GENERAL** CLASS of SYSTEMS
  - **ANALYZE** the SYSTEM
    - TOOLS: TIME-DOMAIN & FREQUENCY-DOMAIN
  - **SYNTHESIZE** the SYSTEM



## D-T SYSTEM EXAMPLES

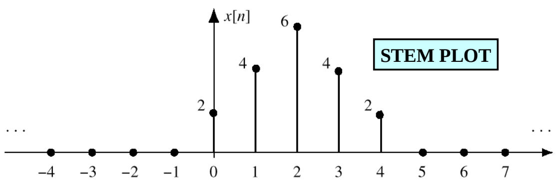



- EXAMPLES:
  - POINTWISE OPERATORS
    - SQUARING: $y[n] = (x[n])^2$
  - RUNNING AVERAGE
    - RULE: "the output at time $n$ is the average of three consecutive input values"



## DISCRETE-TIME SIGNAL

- $x[n]$ is a LIST of NUMBERS
  - INDEXED by "$n$"



# 3-PT AVERAGE SYSTEM

- ADD 3 CONSECUTIVE NUMBERS
  - Do this for each "$n$"

the following input–output equation **Make a TABLE**

$$y[n] = \tfrac{1}{3}(x[n] + x[n+1] + x[n+2])$$

| $n$ | $n < -2$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 | 5 | $n > 5$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $x[n]$ | 0 | 0 | 0 | 2 | 4 | **6** | **4** | **2** | 0 | 0 |
| $y[n]$ | 0 | $\frac{2}{3}$ | 2 | 4 | $\frac{14}{3}$ | **4** | 2 | $\frac{2}{3}$ | 0 | 0 |

**n=0** $y[0] = \frac{1}{3}(x[0] + x[1] + x[2])$

**n=1** $y[1] = \frac{1}{3}(x[1] + x[2] + x[3])$

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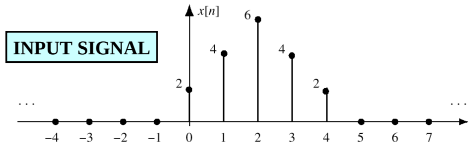


**Figure 5.2** Finite-length input signal, $x[n]$.

$$y[n] = \tfrac{1}{3}(x[n] + x[n+1] + x[n+2])$$

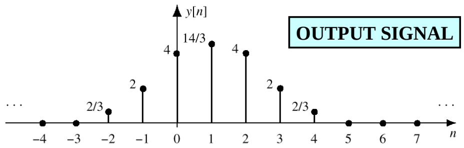


**Figure 5.3** Output of running average, $y[n]$.

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# PAST, PRESENT, FUTURE

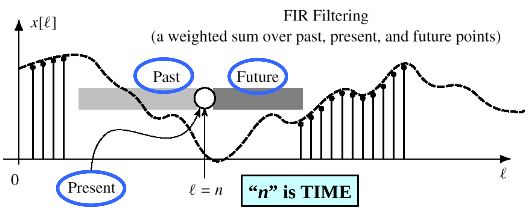


**Figure 5.4** The running-average filter calculation at time index $n$ uses values within a sliding window (shaded). Dark shading indicates the future ($\ell > n$); light shading, the past ($\ell < n$).

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# ANOTHER 3-pt AVERAGER

- Uses "PAST" VALUES of $x[n]$
  - IMPORTANT IF "$n$" represents REAL TIME
    - WHEN $x[n]$ & $y[n]$ ARE STREAMS

$$y[n] = \tfrac{1}{3}(x[n] + x[n-1] + x[n-2])$$

| $n$ | $n < -2$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | $n > 7$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x[n]$ | 0 | 0 | 0 | 2 | 4 | **6** | **4** | **2** | 0 | 0 | 0 | 0 |
| $y[n]$ | 0 | 0 | 0 | $\frac{2}{3}$ | 2 | 4 | $\frac{14}{3}$ | **4** | 2 | $\frac{2}{3}$ | 0 | 0 |

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# GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$
  - DEFINE THE FILTER

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

  - For example, $b_k = \{3, -1, 2, 1\}$

$$y[n] = \sum_{k=0}^{3} b_k x[n-k]$$
$$= 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

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# GENERAL FIR FILTER

- FILTER COEFFICIENTS $\{b_k\}$

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

- FILTER **ORDER** is $M$
- FILTER **LENGTH** is $L = M+1$
  - NUMBER of FILTER COEFFS is $L$

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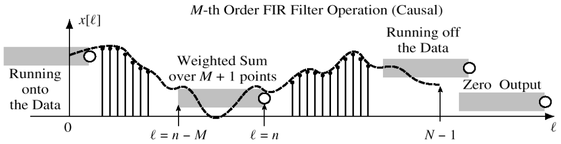
# GENERAL FIR FILTER

- SLIDE a WINDOW across $x[n]$

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

*M*-th Order FIR Filter Operation (Causal)






# FILTERED STOCK SIGNAL

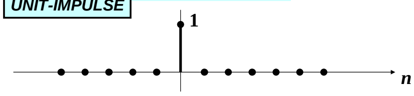



# SPECIAL INPUT SIGNALS

- $x[n]$ = SINUSOID
- $x[n]$ has only one NO FREQUENCY RESPONSE (LATER)

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

UNIT-IMPULSE






# UNIT IMPULSE SIGNAL $\delta[n]$

| $n$ | ... | −2 | −1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | ... |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\delta[n]$ | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\delta[n-3]$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |

$n = 3$

**δ[n] is NON-ZERO When its argument is equal to ZERO**

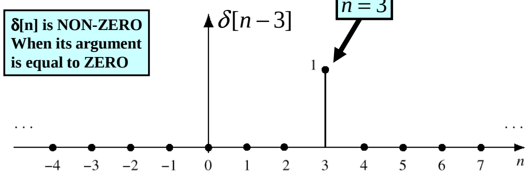


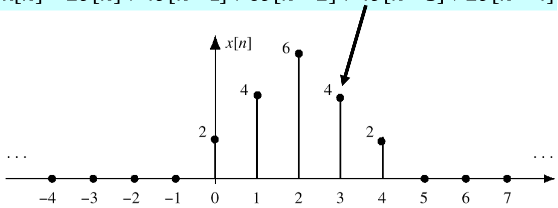

**Figure 5.7** Shifted impulse sequence, $\delta[n-3]$.



# MATH FORMULA for $x[n]$

- Use SHIFTED IMPULSES to write $x[n]$

$$x[n] = 2\delta[n] + 4\delta[n-1] + 6\delta[n-2] + 4\delta[n-3] + 2\delta[n-4]$$






# SUM of SHIFTED IMPULSES

| $n$ | ... | −2 | −1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | ... |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $2\delta[n]$ | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $4\delta[n-1]$ | 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| $6\delta[n-2]$ | 0 | 0 | 0 | 0 | 0 | 6 | 0 | 0 | 0 | 0 | 0 |
| $4\delta[n-3]$ | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 |
| $2\delta[n-4]$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 |
| $x[n]$ | 0 | 0 | 0 | 2 | 4 | 6 | 4 | 2 | 0 | 0 | 0 |

$$x[n] = \sum_k x[k]\delta[n-k]$$ ← **This formula ALWAYS works**

$$= \ldots + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \ldots \quad (5.3.6)$$

## 4-pt AVERAGER

- CAUSAL SYSTEM: USE PAST VALUES
  $$y[n] = \tfrac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$
- INPUT = UNIT IMPULSE SIGNAL = **δ[n]**
  $$x[n] = \delta[n]$$
  $$y[n] = \tfrac{1}{4}\delta[n] + \tfrac{1}{4}\delta[n-1] + \tfrac{1}{4}\delta[n-2] + \tfrac{1}{4}\delta[n-3]$$
- OUTPUT is called "IMPULSE RESPONSE"
  $$h[n] = \{\ldots, 0, 0, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, 0, 0, \ldots\}$$

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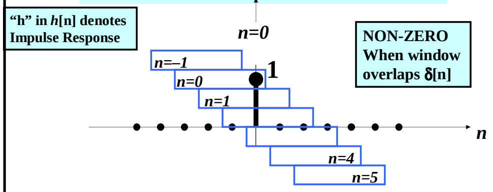
## 4-pt Avg Impulse Response

$$y[n] = \tfrac{1}{4}(x[n] + x[n-1] + x[n-2] + x[n-3])$$

δ[n] "READS OUT" the FILTER COEFFICIENTS

$$h[n] = \{\ldots, 0, 0, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{4}, 0, 0, \ldots\}$$

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## FIR IMPULSE RESPONSE

- Convolution = Filter Definition
  - Filter Coeffs = Impulse Response

| $n$ | $n<0$ | 0 | 1 | 2 | 3 | ... | $M$ | $M+1$ | $n>M+1$ |
|---|---|---|---|---|---|---|---|---|---|
| $x[n]=\delta[n]$ | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $y[n]=h[n]$ | 0 | $b_0$ | $b_1$ | $b_2$ | $b_3$ | ... | $b_M$ | 0 | 0 |

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k]$$

**CONVOLUTION**

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## FILTERING EXAMPLE

- 7-point AVERAGER
  - Removes cosine
    - By making its amplitude (A) smaller

$$y_7[n] = \sum_{k=0}^{6} \left(\tfrac{1}{7}\right) x[n-k]$$

- 3-point AVERAGER
  - Changes A slightly

$$y_3[n] = \sum_{k=0}^{2} \left(\tfrac{1}{3}\right) x[n-k]$$

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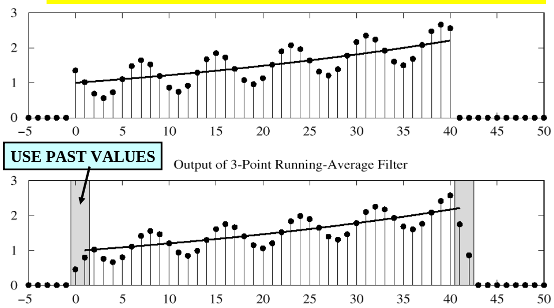
## 3-pt AVG EXAMPLE

Input : $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$ for $0 \le n \le 40$




## 7-pt FIR EXAMPLE (AVG)

Input : $x[n] = (1.02)^n + \cos(2\pi n/8 + \pi/4)$ for $0 \le n \le 40$