Digital Signal Processing

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Digital Signal Processing

Lecture 9

D-to-A Conversion

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License Info for SPFirst Slides

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READING ASSIGNMENTS

- This Lecture:
  - Chapter 4: Sections 4-4, 4-5
- Other Reading:
  - Recitation: Section 4-3 (Strobe Demo)
  - Next Lecture: Chapter 5 (beginning)

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LECTURE OBJECTIVES

- FOLDING: a type of ALIASING
- DIGITAL-to-ANALOG CONVERSION is
  - Reconstruction from samples
    - SAMPLING THEOREM applies
  - Smooth **Interpolation**
- Mathematical Model of D-to-A
  - SUM of SHIFTED PULSES
    - Linear Interpolation example

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SIGNAL TYPES

- A-to-D
  - Convert $x(t)$ to numbers stored in memory
- D-to-A
  - Convert $y[n]$ back to a "continuous-time" signal, $y(t)$
  - $y[n]$ is called a "discrete-time" signal

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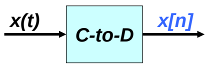
SAMPLING $x(t)$

- UNIFORM SAMPLING at $t = nT_s$
  - IDEAL: $x[n] = x(nT_s)$



Shannon Sampling Theorem

A continuous-time signal $x(t)$ with frequencies no higher than f_{max} can be reconstructed exactly from its samples $x[n] = x(nT_s)$, if the samples are taken at a rate $f_s = 1/T_s$ that is greater than $2f_{max}$.

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NYQUIST RATE

- "Nyquist Rate" Sampling
  - f_s > **TWICE** the HIGHEST Frequency in $x(t)$
  - "Sampling above the Nyquist rate"
- **BANDLIMITED SIGNALS**
  - DEF: x(t) has a HIGHEST FREQUENCY COMPONENT in its SPECTRUM
  - NON-BANDLIMITED EXAMPLE
    - TRIANGLE WAVE is NOT BANDLIMITED

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SPECTRUM for $x[n]$

- INCLUDE ALL SPECTRUM LINES
  - ALIASES
    - ADD INTEGER MULTIPLES of 2π and -2π
  - FOLDED ALIASES
    - ALIASES of NEGATIVE FREQS
- PLOT versus NORMALIZED FREQUENCY
  - i.e., DIVIDE f_o by f_s

$$\hat{\omega} = 2\pi \frac{f}{f_s} + 2\pi\ell$$

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EXAMPLE: SPECTRUM

- $x[n] = A\cos(0.2\pi n + \phi)$
- FREQS @ 0.2π and -0.2π
- ALIASES:
  - $\{2.2\pi, 4.2\pi, 6.2\pi, \ldots\}$ & $\{-1.8\pi, -3.8\pi, \ldots\}$
  - EX: $x[n] = A\cos(4.2\pi n + \phi)$
- ALIASES of NEGATIVE FREQ:
  - $\{1.8\pi, 3.8\pi, 5.8\pi, \ldots\}$ & $\{-2.2\pi, -4.2\pi \ldots\}$

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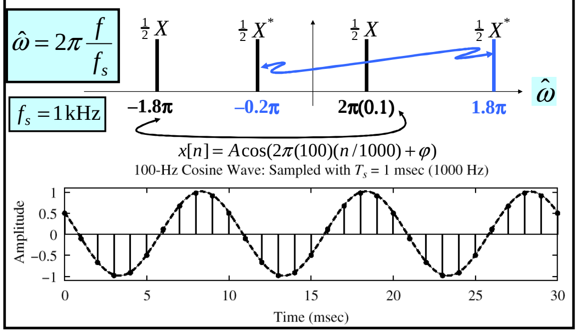
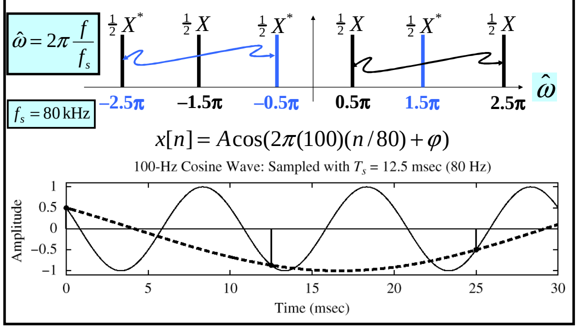
SPECTRUM (MORE LINES)



SPECTRUM (ALIASING CASE)



FOLDING (a type of ALIASING)

- EXAMPLE: 3 different $x(t)$; same $x[n]$

$$\hat{\omega} = 2\pi \frac{100}{1000} = 2\pi(0.1)$$

$$f_s = 1000$$
$$\cos(2\pi(100)t) \rightarrow \cos[2\pi(0.1)n]$$
$$\cos(2\pi(1100)t) \rightarrow \cos[2\pi(1.1)n] = \cos[2\pi(0.1)n]$$
$$\cos(2\pi(900)t) \rightarrow \cos[2\pi(0.9)n]$$
$$= \cos[2\pi(0.9)n - 2\pi n] = \cos[2\pi(-0.1)n] = \cos[2\pi(0.1)n]$$

- 900 Hz “folds” to 100 Hz when f_s=1kHz

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DIGITAL FREQ $\hat{\omega}$ AGAIN

Normalized Radian Frequency

$$\hat{\omega} = \omega T_s = \frac{2\pi f}{f_s} + 2\pi\ell$$ ALIASING

$$\hat{\omega} = \omega T_s = -\frac{2\pi f}{f_s} + 2\pi\ell$$ FOLDED ALIAS

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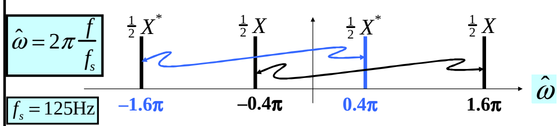
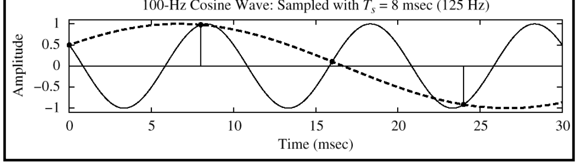
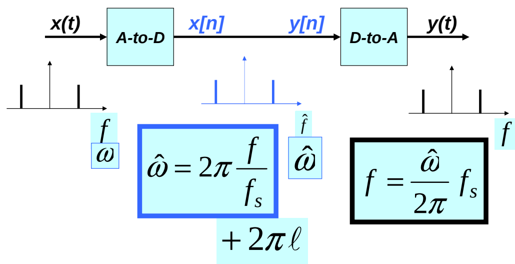
SPECTRUM (FOLDING CASE)



$$x[n] = A\cos(2\pi(100)(n/125) + \varphi)$$



FREQUENCY DOMAINS



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DEMOS from CHAPTER 4

- CD-ROM DEMOS
- SAMPLING DEMO (**con2dis GUI**)
  - Different Sampling Rates
    - Aliasing of a Sinusoid
- STROBE DEMO
  - Synthetic vs. Real
  - Television SAMPLES at 30 fps
- Sampling & Reconstruction

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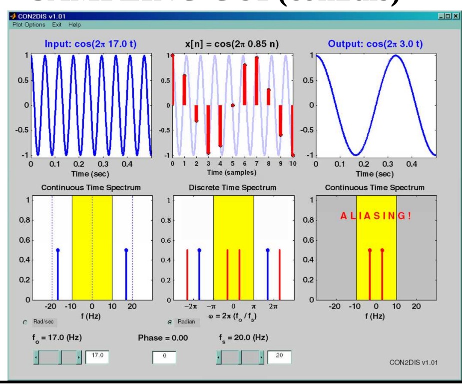
SAMPLING GUI (con2dis)



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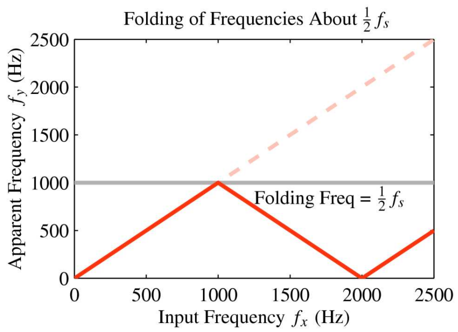
FOLDING DIAGRAM



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D-to-A Reconstruction

- Create continuous $y(t)$ from $y[n]$
  - **IDEAL**
    - If you have formula for $y[n]$
  - Replace n in $y[n]$ with $f_s t$
  - $y[n] = A\cos(0.2\pi n+\phi)$ with $f_s = 8000$ Hz
  - $y(t) = A\cos(2\pi(800)t+\phi)$

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D-to-A is AMBIGUOUS !

- ALIASING
  - Given $y[n]$, which $y(t)$ do we pick ? ? ?
  - INFINITE NUMBER of $y(t)$
    - PASSING THRU THE SAMPLES, $y[n]$
  - D-to-A RECONSTRUCTION MUST CHOOSE ONE OUTPUT
- RECONSTRUCT THE SMOOTHEST ONE
  - THE LOWEST FREQ, if $y[n]$ = sinusoid

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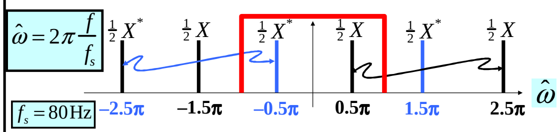
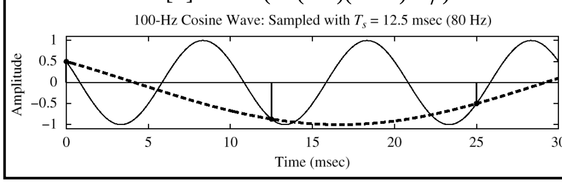
SPECTRUM (ALIASING CASE)



$x[n] = A\cos(2\pi(100)(n/80)+\varphi)$



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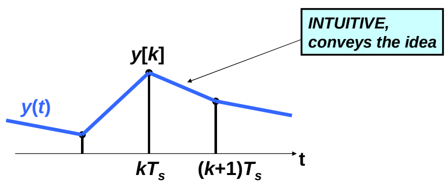
Reconstruction (D-to-A)

- CONVERT STREAM of NUMBERS to $x(t)$
- "CONNECT THE DOTS"
- INTERPOLATION



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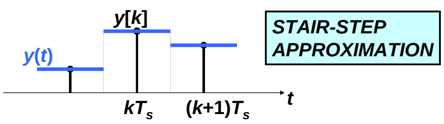
SAMPLE & HOLD DEVICE

- CONVERT $y[n]$ to $y(t)$
  - $y[k]$ should be the value of $y(t)$ at $t = kT_s$
  - Make $y(t)$ equal to $y[k]$ for
    - $kT_s - 0.5T_s < t < kT_s + 0.5T_s$



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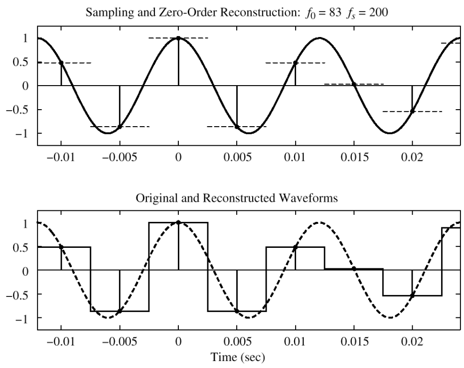
SQUARE PULSE CASE



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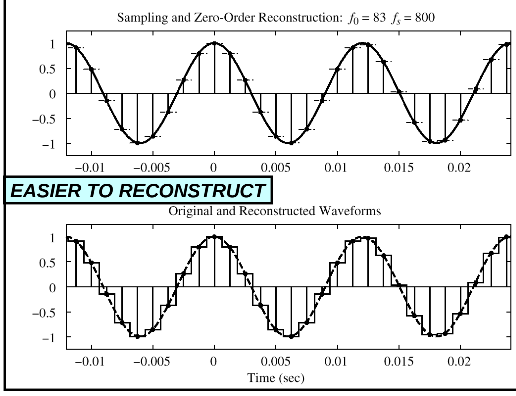
OVER-SAMPLING CASE



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MATH MODEL for D-to-A

$$y(t) = \sum_{n=-\infty}^{\infty} y[n]p(t - nT_s)$$

SQUARE PULSE:

$$p(t) = \begin{cases} 1 & -\frac{1}{2}T_s < t \le \frac{1}{2}T_s \\ 0 & \text{otherwise} \end{cases}$$

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EXPAND the SUMMATION

$$\sum_{n=-\infty}^{\infty} y[n]p(t - nT_s) =$$

$$\ldots + y[0]p(t) + y[1]p(t - T_s) + y[2]p(t - 2T_s) + \ldots$$

- SUM of SHIFTED PULSES $p(t\text{-}nT_s)$
  - "WEIGHTED" by $y[n]$
  - CENTERED at $t=nT_s$
  - SPACED by T_s
    - RESTORES "REAL TIME"

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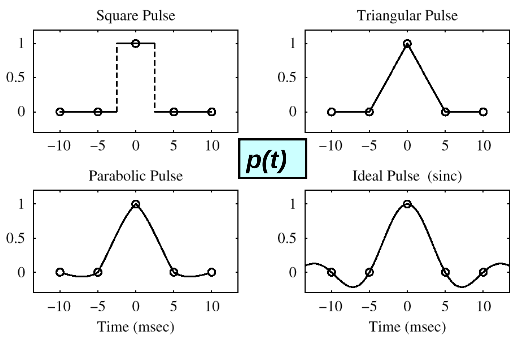

Figure 4.17 Four different pulses for D-to-C conversion. The sampling period is $T_s = 0.005$, i.e., $f_s = 200$ Hz. Note that the duration of each pulse is approximately one or two times T_s.

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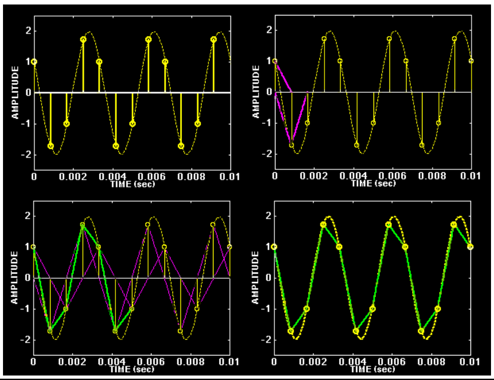
TRIANGULAR PULSE (2X)



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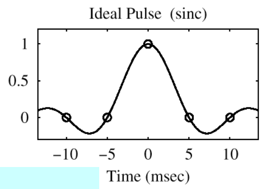
OPTIMAL PULSE ?

CALLED "BANDLIMITED INTERPOLATION"



$$p(t) = \frac{\sin \frac{\pi t}{T_s}}{\frac{\pi t}{T_s}} \quad \text{for} -\infty < t < \infty$$

$$p(t) = 0 \quad \text{for} \;\; t = \pm T_s, \pm 2T_s, \ldots$$

FOLDING DIAGRAM

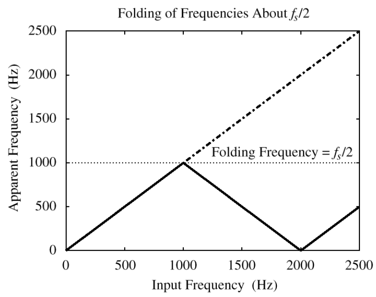
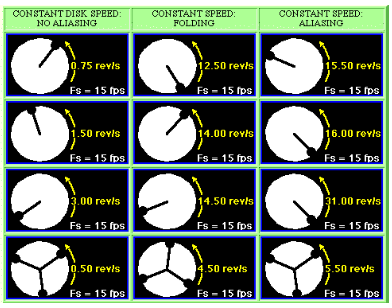


Figure 4.4 Folding of a sinusoid sampled at $f_s = 2000$ samples/sec. The apparent frequency is the lowest frequency of a sinusoid that has exactly the same samples as the input sinusoid.

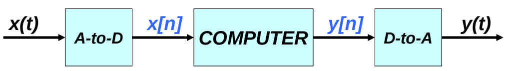
STROBE DEMO (Synthetic)



ALIASING & FOLDING

- $x(t)$ = SINUSOID @ f_o
- SAMPLED SIGNAL: $x[n] = x(n/f_s)$
- **ALIASING**:
  - $x[n]$ COULD HAVE COME FROM
  - $(f_o + f_s)$
  - or $(f_o - f_s)$
  - or $(f_o + 2f_s)$
  - or $(f_o - 2f_s)$, etc.
- **FOLDING**:
  - A type of ALIASING
  - $x[n]$ COULD BE FROM:
  - $(-f_o + f_s)$
  - or $(-f_o - f_s)$
  - or $(-f_o + 2f_s)$
  - or $(-f_o - 2f_s)$, etc.

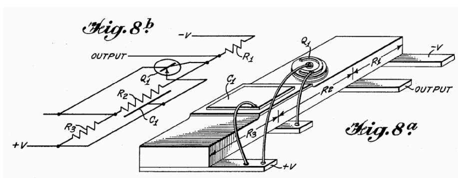
D-to-A Reconstruction



- Create continuous $y(t)$ from $y[n]$
  - **REALISTIC CONSTRAINT:** SMOOTH $y(t)$
    - Use the lowest possible frequency
  - $y[n]$ is a list of numbers
  - How fast?
  - In MATLAB: `soundsc(yy,fs)`

INVENTION of the IC

- Integrated Circuit: 1959 by Jack Kilby
  - http://www.eepatents.com/feature/



FOUR FREQUENCY AXES

- ANALOG FREQUENCY: f, ω
- DIGITAL FREQUENCY

Normalized Radian Frequency

$$\hat{\omega} = \omega T_s$$

$$\hat{\omega} = 2\pi \frac{f}{f_s}$$

Normalized Cyclic Frequency

$$\hat{f} = \hat{\omega}/(2\pi) = f T_s = f/f_s$$

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FOLDING DERIVATION

- Negative Freqs can give the same $\hat{\omega}$

$$x(t) = A\cos(2\pi(-f + \ell f_s)t - \varphi)$$

$$x[n] = x(nT_s) = A\cos(2\pi(-f + \ell f_s)nT_s - \varphi)$$

$$x[n] = A\cos((-2\pi f T_s)n + (2\pi \ell f_s T_s)n - \varphi)$$

$$x[n] = A\cos((2\pi f T_s)n - 2\pi \ell n + \varphi)$$

$\cos(-\theta) = \cos\theta$

$$x[n] = A\cos(\hat{\omega} n + \varphi)$$

SAME DIGITAL SIGNAL

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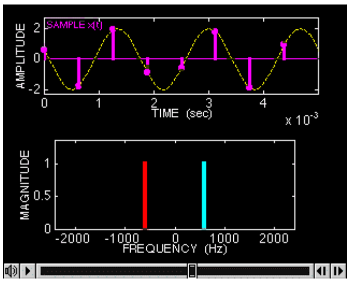
SAMPLING DEMO (Ch. 4)



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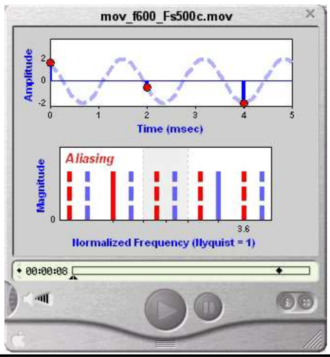
SAMPLING DEMO (Ch. 4)



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