Digital Signal Processing

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Digital Signal Processing

Lecture 1

Sinusoids

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READING ASSIGNMENTS

- This Lecture:
  - Chapter 2, pp. 9-17
- Appendix A: Complex Numbers
- Appendix B: MATLAB
- Chapter 1: Introduction

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LECTURE OBJECTIVES

- Write general formula for a "sinusoidal" waveform, or signal
- From the formula, plot the sinusoid versus time
- What's a **signal**?
  - It's a **function** of time, x(t)
  - in the mathematical sense

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What's a signal

- A signal can be defined as
  - a pattern of variations of a physical quantity that can be manipulated, stored, or transmitted by physical process.
- In the mathematical sense it's a **function** of time, $x(t)$, that carries an information.

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TUNING FORK EXAMPLE

- CD-ROM demo
- "A" is at 440 Hertz (Hz)
- Waveform is a SINUSOIDAL SIGNAL
- Computer plot looks like a sine wave
- This should be the mathematical formula:

$$A\cos(2\pi(440)t + \varphi)$$

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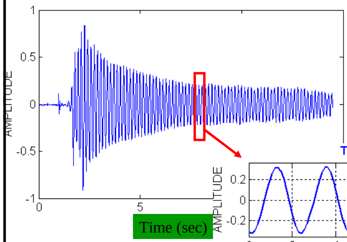
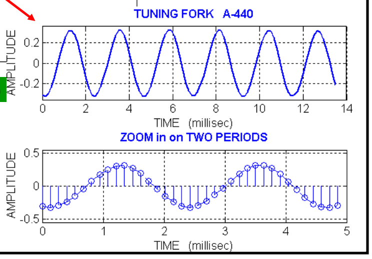
TUNING FORK A-440 Waveform



$T \approx 8.15 - 5.85$
$= 2.3\,\text{ms}$



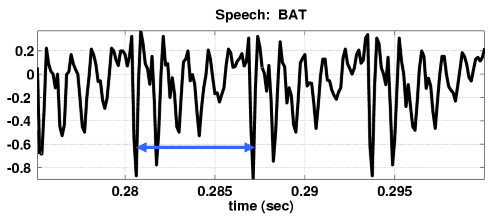
$f = 1/T$
$= 1000/2.3$
$\approx 435\,\text{Hz}$

SPEECH EXAMPLE

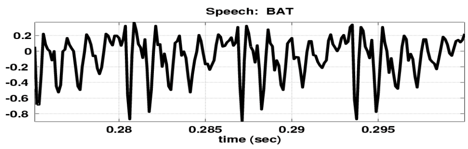
- More complicated signal (BAT.WAV)
- Waveform ***x*(*t*)** is NOT a Sinusoid
- Theory will tell us
  - ***x*(*t*)** is approximately a sum of sinusoids
  - FOURIER ANALYSIS
    - Break ***x*(*t*)** into its sinusoidal components
  - Called the FREQUENCY SPECTRUM

Speech Signal: BAT

- Nearly **<u>Periodic</u>** in Vowel Region
  - Period is (Approximately) T = 0.0065 sec



One-dimensional continuous-time signal

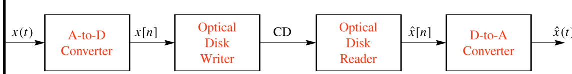


- This speech signal is an example of one-dimensional continuous-time signal.
- Can be represented mathematically as a function of single independent variable (t).

Two-dimensional stationary signal

- This is a two dimensional signal (an image)
- A spatial pattern not varying in time
- Represented mathematically as a function of two spatial variables (x,y)
- However, videos are time-varying images that involves three independent variables (x,y,t)

- Block diagram of an audio CD



McClellan, Schafer and Yoder, *Signal Processing First*, ISBN 0-13-065562-7.
Pearson Prentice Hall, Inc. Upper Saddle River, NJ 07458. © 2003

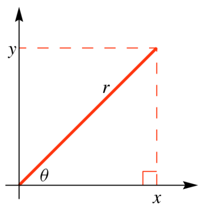
DIGITIZE the WAVEFORM

- ***x*[*n*]** is a SAMPLED SINUSOID
  - A list of numbers stored in memory
- Sample at 11,025 samples per second
  - Called the SAMPLING RATE of the A/D
  - Time between samples is
    - 1/11025 = 90.7 microsec
- Output via D/A hardware (at F_{samp})

STORING DIGITAL SOUND

- ***x*[*n*]** is a SAMPLED SINUSOID
  - A list of numbers stored in memory
- CD rate is 44,100 samples per second
- 16-bit samples
- Stereo uses 2 channels
- Number of bytes for 1 minute is
  - 2 X (16/8) X 60 X 44100 = 10.584 Mbytes

SINE and COSINE functions



$$\sin\theta = \frac{y}{r} \implies y = r\sin\theta$$

$$\cos\theta = \frac{x}{r} \implies x = r\cos\theta$$

SINES and COSINES

- Always use the COSINE FORM

$$A\cos(2\pi(440)t + \varphi)$$

- Sine is a special case:

$$\sin(\omega t) = \cos(\omega t - \tfrac{\pi}{2})$$

SINUSOIDAL SIGNAL

$$A\cos(\omega t + \varphi)$$

- **FREQUENCY** ω
  - Radians/sec
  - Hertz (cycles/sec)

$$\omega = (2\pi)f$$

- **PERIOD** (in sec)

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

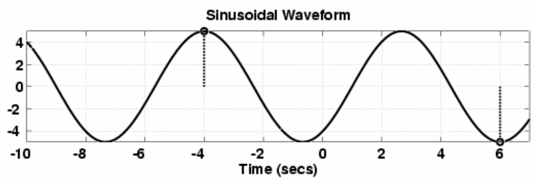
- **AMPLITUDE** A
  - Magnitude
- **PHASE** φ

EXAMPLE of SINUSOID

- Given the Formula

$$5\cos(0.3\pi t + 1.2\pi)$$

- Make a plot



PLOT COSINE SIGNAL

$$5\cos(0.3\pi t + 1.2\pi)$$

- Formula defines A, ω, and φ

$$A = 5$$
$$\omega = 0.3\pi$$
$$\varphi = 1.2\pi$$

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PLOTTING COSINE SIGNAL from the FORMULA

$$5\cos(0.3\pi t + 1.2\pi)$$

- Determine **period**:

$$T = 2\pi/\omega = 2\pi/0.3\pi = 20/3$$

- Determine a **peak** location by solving

$$(\omega t + \varphi) = 0 \quad \Rightarrow (0.3\pi t + 1.2\pi) = 0$$

- **Zero** crossing is T/4 before or after
- Positive & Negative peaks spaced by T/2

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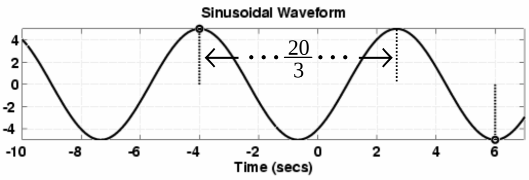
PLOT the SINUSOID

$$5\cos(0.3\pi t + 1.2\pi)$$

- Use T=20/3 and the peak location at t=-4



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Basic properties of the sine and cosine functions

Property	Equation
Equivalence	$\sin\theta = \cos(\theta - \pi/2)$ or $\cos(\theta) = \sin(\theta + \pi/2)$
Periodicity	$\cos(\theta + 2\pi k) = \cos\theta$, when k is an integer
Evenness of cosine	$\cos(-\theta) = \cos\theta$
Oddness of sine	$\sin(-\theta) = -\sin\theta$
Zeros of sine	$\sin(\pi k) = 0$, when k is an integer
Ones of cosine	$\cos(2\pi k) = 1$ when k is an integer.
Minus ones of cosine	$\cos[2\pi(k + \frac{1}{2})] = -1$, when k is an integer.

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Some basic trigonometric identities

Number	Equation
1	$\sin^2\theta + \cos^2\theta = 1$
2	$\cos 2\theta = \cos^2\theta - \sin^2\theta$
3	$\sin 2\theta = 2\sin\theta\cos\theta$
4	$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
5	$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

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Sampling and plotting sinusoids

- Plot the following function

$$20\cos(2\pi(40)t - 0.4\pi)$$

- Must evaluate $x(t)$ at a discrete set of times, $t_n = nT_s$, where n is an integer

$$x(nT_s) = 20\cos(80\pi n T_s - 0.4\pi)$$

- T_s is called sample spacing or sampling period
- Matlab program

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