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Vize 2 (29.12.2009)

**0113620 – Sayısal İşaret İşleme**

S1. (a) "Doğrusal (lineer) sistem" nedir? (5)

(b) "Zamanla değişmeyen (time-invariant) sistem" nedir? (5)

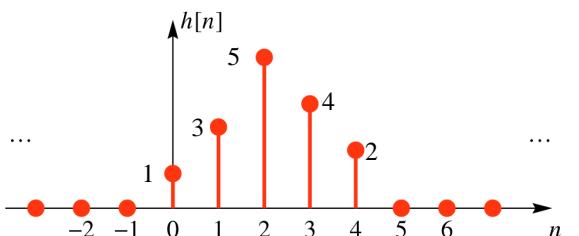
(c) "Nedensel (causal) sistem" nedir? (5)  
tanımlayınız.

S2. Bir FIR sayısal süzgeçin girişine bir darbe fonksiyonu ( $x[n]=\delta[n]$ ) uygulandığında şeklindeki süzgeç cevabı ( $h[n]$ ) elde edilmektedir.

(a) Bu süzgeçin fark denklemine ait süzgeç katsayılarını ( $\{b_k\}$ ) belirleyiniz. (10)

(b)  $x[n]=\delta[n] - \delta[n-1] + \delta[n-2]$  işaretin süzgeç girişine uygulandığında çıkış ( $y[n]$ ) bulunuz. (10)

(c)  $y[n]$  yi çiziniz. (5)



S3. Doğrusal zamanla değişmeyen (lineer time-invariant) bir filtre  $y[n]=x[n] + 2x[n-1] + x[n-2]$  gibi bir fark denklemi ile belirlenmektedir. Buna göre;

(a) Bu sistemin frekans cevabını bulunuz. (7)

(b) Frekans cevabının genlik ve fazını çiziniz. (8)

(c) Giriş  $x[n]=10+4\cos(0.5\pi n+\pi/4)$  olduğunda çıkışı bulunuz. (10)

(d) Giriş birim darbe fonksiyonu ( $\delta[n]$ ) olduğunda çıkışı bulunuz. (5)

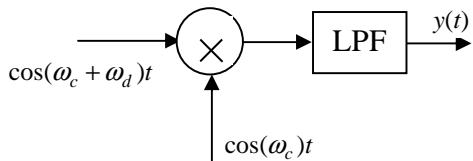
(e) Sistem fonksiyonu  $H(z)$  yi polinom olarak bulunuz. (5)

(f) Sistemin 0 ve kutuplarını bulunuz. (5)

S4. Aşağıda verilen birinci dereceden (first order) sisteme ait darbe cevabını (impulse response,  $h[n]$ ) bulunuz. (10)

$$y[n]=0.5y[n-1] + 4x[n-3]$$

S5. Aşağıdaki sisteme alçak geçiren süzgeçin kesim frekansı  $\omega_c$  olduğuna göre sistemin çıkışı  $y(t)$  ne olur. (10)



$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

$$e^{-j\omega t} = \cos(\omega t) - j \sin(\omega t)$$

## CEVAPLAR

C1. (a)

A system is said to be linear if it meets the following two criteria:

1. Scaling: If input  $x$  to the system results in output  $X$ , then an input of  $2x$  will produce output of  $2X$  (the magnitude of the system output is proportional to the magnitude of the system input)
2. Superposition: If input  $x$  produces output  $X$ , and input  $y$  produces output  $Y$ , then an input of  $x + y$  will produce an output of  $X + Y$  (the system handles two simultaneous inputs independently, and they do not interact within the system).

(b)

A time-invariant system is one whose output does not depend explicitly on time. Time-Shifting the input will cause the same time-shift in the output

(c)

A causal system is a system where the output depends on past and/or current inputs but not future inputs.

C2.

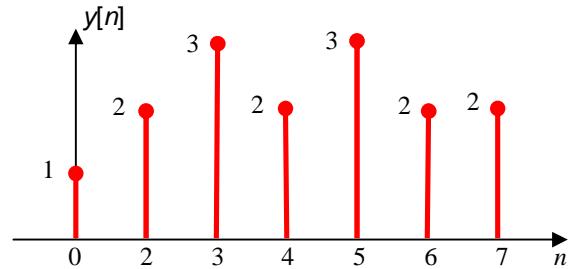
(a)

$$\{b_k\} = \{1, 3, 5, 4, 2\}$$

$$(b) \quad y[n] = h[n] * x[n]$$

(c)

$$\begin{array}{ccccccccc} h[n] & 1 & 3 & 5 & 4 & 2 \\ \hline x[n] & 1 & -1 & 1 & & & & & \\ \hline & 1 & 3 & 5 & 4 & 2 & & & \\ & & -1 & -3 & -5 & -4 & -2 & & \\ \hline & & 1 & 3 & 5 & 4 & 2 & & \\ y[n] & 1 & 2 & 3 & 2 & 3 & 2 & & 2 \end{array}$$



C3.

$$y[n] = x[n] + 2x[n-1] + x[n-2]$$

(a) Filter coefficients are  $\{b_k\} = \{1, 2, 1\}$ .

$$H(\hat{\omega}) = 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

(b)

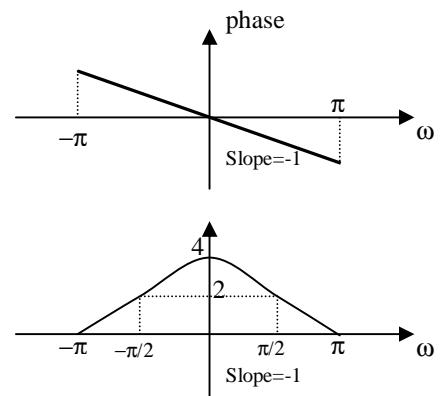
$$H(\hat{\omega}) = e^{-j\hat{\omega}}(e^{j\hat{\omega}} + 2 + e^{-j\hat{\omega}}) = e^{-j\hat{\omega}}(2 + 2\cos\hat{\omega})$$

$$\text{Phase} = -\hat{\omega} \quad \text{Magnitude} = 2 + 2\cos\hat{\omega}$$

$$|H| = 4 \quad \text{at} \quad \hat{\omega} = 0$$

$$|H| = 2 \quad \text{at} \quad \hat{\omega} = \frac{\pi}{2}$$

$$|H| = 0 \quad \text{at} \quad \hat{\omega} = \pi$$



(c)

$$x[n] = 10 + 4\cos(\frac{\pi}{2}n + \frac{\pi}{4})$$

$$= 10 + 2e^{j\frac{\pi}{4}}e^{j\frac{\pi}{2}n} + 2e^{-j\frac{\pi}{4}}e^{-j\frac{\pi}{2}n}$$

$$y[n] = 10H(0) + H(\frac{\pi}{2})2e^{j\frac{\pi}{4}}e^{j\frac{\pi}{2}n} + H(-\frac{\pi}{2})2e^{-j\frac{\pi}{4}}e^{-j\frac{\pi}{2}n}$$

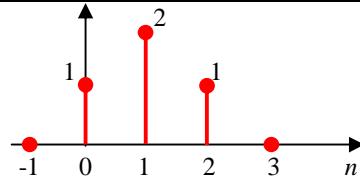
$$H(0) = 4e^{j0} \quad H(\frac{\pi}{2}) = 2e^{-j\frac{\pi}{2}} \quad H(-\frac{\pi}{2}) = 2e^{j\frac{\pi}{2}}$$

$$y[n] = 40 + 4e^{-j\frac{\pi}{2}}e^{j\frac{\pi}{4}}e^{j\frac{\pi}{2}n} + 4e^{j\frac{\pi}{2}}e^{-j\frac{\pi}{4}}e^{-j\frac{\pi}{2}n}$$

$$= 40 + 8\cos(\frac{\pi}{2}n - \frac{\pi}{4})$$

(d)

$$x[n] = \delta[n] \rightarrow y[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$$



(e)

$$\begin{aligned} H(z) &= 1 + 2z^{-1} + z^{-2} \\ &= z^{-2}(z^2 + 2z + 1) \\ &= \frac{z^2 + 2z + 1}{z^2} \end{aligned}$$

(f)

$$\begin{aligned} H(z) &= \frac{z^2 + 2z + 1}{z^2} \\ &= \frac{(z+1)(z+1)}{zz} \end{aligned}$$

2 zeros in -1 and 2 poles in 0

C4.

Darbe cevabı:  $h[n] = 0.5h[n-1] + 4\delta[n-3]$ ,  
 $n = 0, 1, 2, 3, \dots$  için  $h[n]$  hesaplanırsa;

$$\begin{aligned} h[0] &= 0.5h[-1] + \delta[-3] = 0, \\ h[1] &= 0.5h[0] + \delta[-2] = 0, \\ h[2] &= 0.5h[1] + \delta[-1] = 0, \\ h[3] &= 0.5h[2] + \delta[0] = 4, \\ h[4] &= 0.5h[3] + \delta[1] = 2, \\ h[5] &= 0.5h[4] + \delta[2] = 1, \\ h[6] &= 0.5h[5] + \delta[3] = 0.5, \\ h[7] &= 0.5h[6] + \delta[4] = 0.25, \\ &\vdots \end{aligned}$$

Alternatif olarak, aşağıdaki gibi de çözülebilir:

$$Y(z) = 0.5z^{-1}Y(z) + 4z^{-3}X(z)$$

$$Y(z)(1 - 0.5z^{-1}) = 4z^{-3}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{4z^{-3}}{1 - 0.5z^{-1}}$$

Ters Z dönüşümü alınırsa;

$$h[n] = (0.5)^n u[n] 4z^{-3} = 4(0.5)^{n-3} u[n-3]$$

$n$  nin 3 ve 3 ten büyük değerleri için  $h[n] = 4(0.5)^{n-3}$  tür.

$$\text{Buna göre;} \quad h[n] = \begin{cases} 0 & n < 3 \\ 4(0.5)^{n-3} & n \geq 3 \end{cases} \quad \text{olur.}$$

C5.

One solution by using Euler equations:

$$\begin{aligned} \cos \omega_c t \cos(\omega_c + \omega_d)t &= \left( \frac{e^{j\omega_c t} + e^{-j\omega_c t}}{2} \right) \left( \frac{e^{j(\omega_c + \omega_d)t} + e^{-j(\omega_c + \omega_d)t}}{2} \right) \\ &= \frac{1}{4} \left\{ e^{j\omega_c t} e^{j(\omega_c + \omega_d)t} + e^{j\omega_c t} e^{-j(\omega_c + \omega_d)t} + e^{-j\omega_c t} e^{j(\omega_c + \omega_d)t} + e^{-j\omega_c t} e^{-j(\omega_c + \omega_d)t} \right\} \\ &= \frac{1}{4} \left\{ e^{j(\omega_c + \omega_d)t} + e^{-j(-\omega_c + \omega_c + \omega_d)t} + e^{j(-\omega_c + \omega_c + \omega_d)t} + e^{-j(\omega_c + \omega_c + \omega_d)t} \right\} \\ &= \frac{1}{4} \left\{ e^{j(2\omega_c + \omega_d)t} + e^{-j(2\omega_c + \omega_d)t} + e^{j\omega_d t} + e^{-j\omega_d t} \right\} \\ &= \frac{1}{2} \{ \cos(2\omega_c + \omega_d)t + \cos \omega_d t \} \end{aligned}$$

Because the cut-off frequency of the low-pass filter is  $\omega_c$ , the signal component with the frequency value of "2 $\omega_c + \omega_d$ " will be removed. Therefore the result is:

$$y(t) = \frac{1}{2} \cos \omega_d t$$