

Digital Audio and Speech Processing (Sayısal Ses ve Konuşma İşleme)

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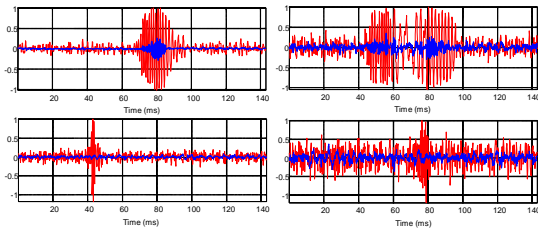
<http://www3.yildiz.edu.tr/~naydin>

Time-Frequency Analysis

Stationary/nonstationary signals

- In real world most signals are highly non-stationary and sometimes last only for a short time.
- Signal analysis methods which assume that the signal is stationary are not appropriate.
- Therefore time-frequency analysis of such signals is necessary.

Some nonstationary signals



Forward (red) and reverse (blue) flow components are shown (after Hilbert transform process)

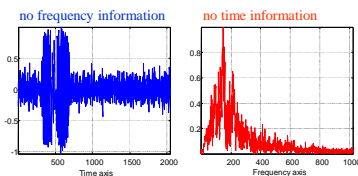
Time-Frequency Analysis...

- The time representation is usually the first description of a signal $s(t)$ obtained by a receiver recording variations with time.
- The frequency representation, which is obtained by the well known Fourier transform (FT), highlights the existence of periodicity,
- and is also a useful way to describe a signal.

Time-Frequency Analysis...

- The relationship between frequency and time representations of a signal can be defined as

$$S(\omega) = \int_{-\infty}^{+\infty} s(t)e^{-j\omega t} dt, \quad s(t) = \int_{-\infty}^{+\infty} S(\omega)e^{j\omega t} d\omega$$

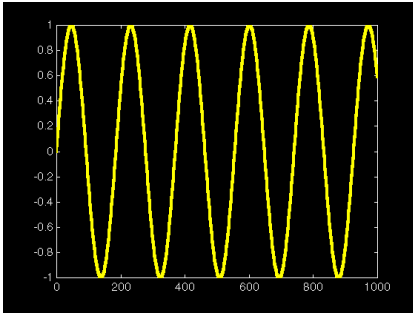


Time-Frequency Analysis

A joint time-frequency representation is necessary to observe evolution of the signal both in time and frequency.

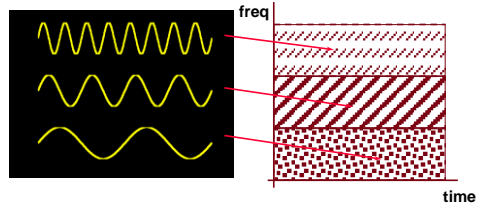
1. Linear methods (Windowed Fourier Transform (WFT), and Wavelet Transform (WT))
 - Decomposes a signal into time-frequency atoms.
 - Computationally efficient
 - Time-frequency resolution trade-off
2. Bilinear (quadratic) methods (Wigner-Ville distribution)
 - Based upon estimating an instantaneous energy distribution using a bilinear operation on the signal.
 - Computationally intense
 - Arbitrarily high resolution in time-and frequency
 - Cross term interference

Harmonic



Fourier Analysis

$$F_s(\omega, t) = \int_{-\infty}^{+\infty} s(t)e^{-j\omega t} dt$$



- Basis functions are smooth, analytic
- Solutions to natural differential equations
- Assumes signal is stationary

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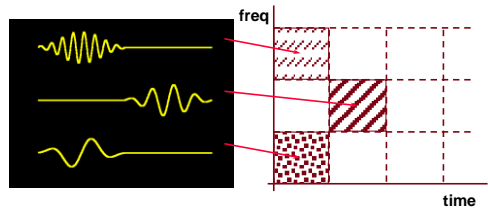
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Windowed Fourier Transform...

$$F_s(\omega, t) = \int_{-\infty}^{+\infty} s(t)g^*(t-\tau)e^{-j\omega t} dt$$

- $g(t)$: short time analysis window localised around $t=0$ and $\omega=0$
- Also called Short-Time Fourier Transform

...Windowed Fourier Transform...



- Windows of fixed size but varying shape
- Fixed size implies fixed time and frequency resolution
- Useful for analysis of narrowband processes

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...Windowed Fourier Transform...

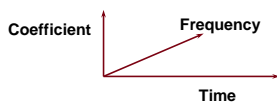
for each **F**requency

for each **T**ime

Coefficient (**F,T**) = $\int_{\text{all time}} \text{Signal} \times \text{window}(\mathbf{F,T})$

end

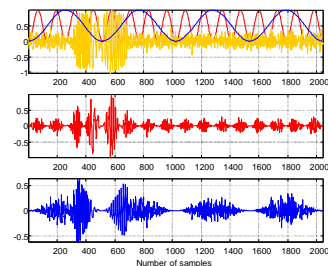
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...Windowed Fourier Transform...

6.12.16

- Segmenting a long signal into smaller sections with 128 point and 512 point Hanning window (critically sampled).



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...Windowed Fourier Transform

- Three important WFT parameters for analysis of a particular signal need to be determined:

- Window type,
- Window size,
- Required overlap ratio

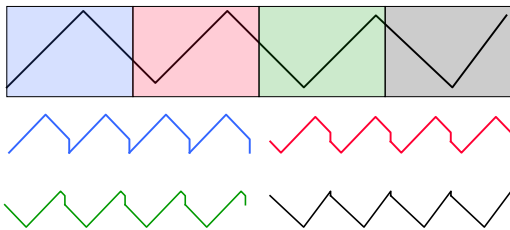
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Window type...

- The FT makes an implicit assumption that the signal within the measured time is repetitive.
- Most real signals will have discontinuities at the ends of the measured time, and
- when the FFT assumes the signal repeats, it will also assume discontinuities that are not there.

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Window type...

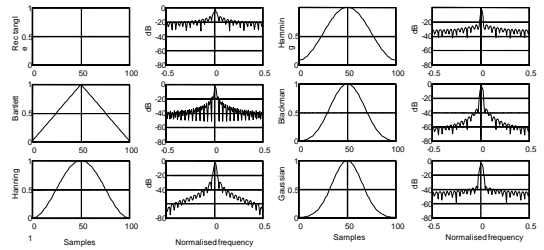


- Discontinuities can be eliminated by multiplying the signal with a window function.

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...Window type

- Some window types and corresponding power spectra



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Window size...

- In a FFT process, there is a well known trade-off between frequency resolution ($\Delta\omega$) and time resolution (Δt), which can be expressed as

$$\Delta t = \frac{W}{\omega_s}, \quad \Delta\omega = \frac{\omega_s}{W}$$

- where W is window length and ω_s is sampling frequency.
- To use the WFT, one has to make a trade-off between time-resolution and frequency resolution.

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...Window size

- If no overlap is employed, processing N_s length data by using W length analysis window will result in a time-frequency distribution having a dimension that almost equals to the dimension of the original signal space (critically sampled WFT).
- The actual dimension of the time-frequency distribution is

$$M = \frac{N_s}{W} \times [W + 1]$$

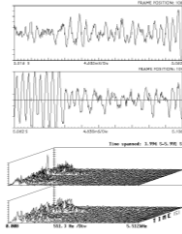
- The best combination of Δt and Δf depends on the signal being processed and best time-frequency resolution trade-off needs to be determined empirically.

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Window overlap ratio...

- A short duration signal may be lost when a windowing function is used prior to the FFT.
- In this case an overlap ratio to some degree must be employed.
- In overlapped WFT, the data frames of length W are processed sequentially by sliding the window ' $W-O_L$ ' times at each processing stage, where O_L is the number of overlapped samples.
- Consequently, overlapping FFT windows produces higher dimensional WFTs.
- In an overlapped WFT process, the dimension of the resultant time-frequency distribution is

$$M = \frac{N_s - O_L}{W - O_L} \times [W + 1]$$



An example of possible embolic signal at the edges of two consecutive frames and related 3d spectrum without a window and with a Hanning window function

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...Window overlap ratio...

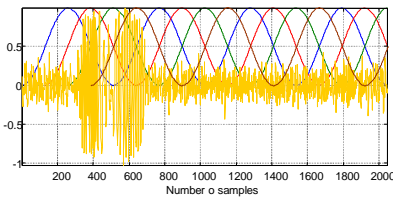
- The overlapping process introduces a predictable time shift on the actual location of a transient event on the time-frequency plane of the FFT.
- The duration of the time shift depends on the overlap ratio used, while the direction of the time shift is dictated by the way that the data are arranged prior to the FFT.
- Duration of the time shift can be estimated as ' $(\text{number of overlapped samples}/2) \times \text{sampling time}$ '.
- The time shift can be adjusted by adding zeros equally at both ends of the original data array.
- In this case the dimension of the overlapped WFT is

$$M = \frac{N_s}{W - O_L} \times [W + 1]$$

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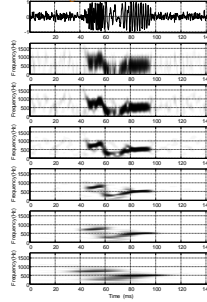
...Window overlap ratio

- Segmenting a long signal into smaller sections with 512 point Hanning window (different overlap strategies).

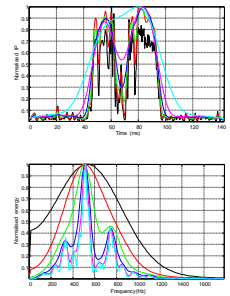


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TFDs with 16, 32, 64, 128, 256, 512 points windowing

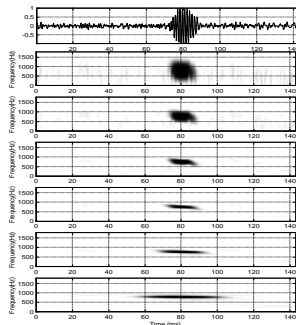


Normalised IP and energy with 16(black), 32(red), 64(green), 128(blue), 256(magenta), 512(cyan) points windowing



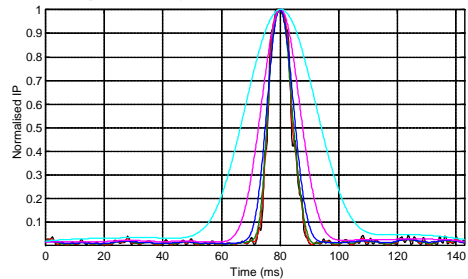
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- TFDs with 16, 32, 64, 128, 256, 512 points Hanning windowing



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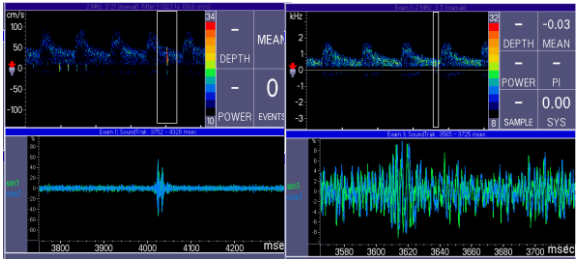
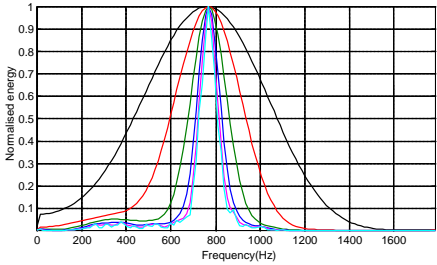
Normalised IP with 16(black), 32(red), 64(green), 128(blue), 256(magenta), 512(cyan) point windowing



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Sonogram

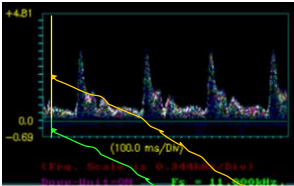
Normalised energy with 16(black), 32(red), 64(green), 128(blue), 256(magenta), 512(cyan) point windowing



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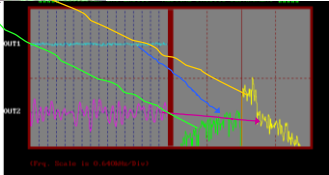
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Sonogram



- + frequencies
- forward flow
- - frequencies
- reverse flow

- Fast Fourier Transform
- Spectrum



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