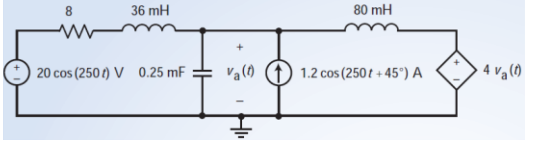
BLM1612 - Circuit Theory

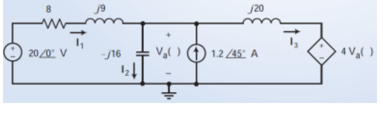
Examples

Example 53…

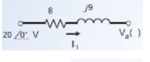
- Determine the voltage $v_a(t)$ for the following circuit



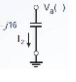
…Example 53…



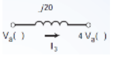
$$\mathbf{I}_1 = \frac{20\angle 0^\circ - \mathbf{V}_a(\omega)}{8 - j9}$$



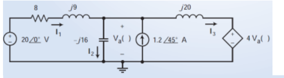
$$\mathbf{I}_2 = \frac{\mathbf{V}_a(\omega) - 0}{-j16} = \frac{\mathbf{V}_a(\omega)}{-j16}$$



$$\mathbf{I}_3 = \frac{\mathbf{V}_a(\omega) - 4\mathbf{V}_a(\omega)}{j20} = -\frac{3\mathbf{V}_a(\omega)}{j20}$$



…Example 53



Applying KCL at the top node of the capacitor gives

$$\mathbf{I}_1 + 1.2\angle 45^\circ = \mathbf{I}_2 + \mathbf{I}_3$$

Substituting for $\mathbf{I}_1$, $\mathbf{I}_2$ and $\mathbf{I}_3$ gives

$$\frac{20\angle 0^\circ - \mathbf{V}_a(\omega)}{8 - j9} + 1.2\angle 45^\circ = \frac{\mathbf{V}_a(\omega)}{-j16} + \left(-\frac{3\mathbf{V}_a(\omega)}{j20}\right)$$

Collecting the terms involving $\mathbf{V}_a(\omega)$ gives

$$\frac{20\angle 0^\circ}{8 - j9} + 1.2\angle 45^\circ = \left(\frac{1}{8 - j9} + \frac{1}{-j16} - \frac{3}{j20}\right)\mathbf{V}_a(\omega)$$

Solving for $\mathbf{V}_a(\omega)$, perhaps using MATLAB (see Figure 10.6-5), gives

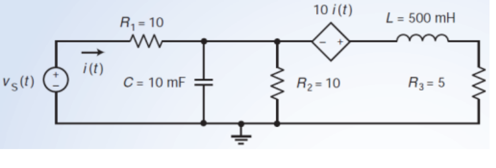
$$\mathbf{V}_a(\omega) = 12.43\angle -81.2^\circ \text{ V}$$

The corresponding sinusoid is

$$v_a(t) = 12.43 \cos(250t - 81.2^\circ) \text{ V}$$

Example 54…

- Input to the circuit is the voltage source voltage $v_a(t) = 10 \cos (10t)$ V.
- The output is the current $i(t)$ in resistor R_1.
- Determine $i(t)$.



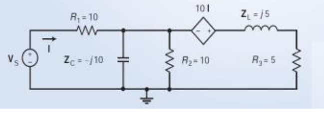
…Example 54…

First, we will represent the circuit in the frequency domain using phasors and impedances. The impedances of the capacitor and inductor are

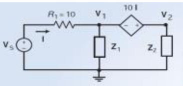
$$\mathbf{Z}_c = -j\frac{1}{10(0.010)} = -j10\,\Omega \text{ and } \mathbf{Z}_L = j10(0.5) = j5\,\Omega$$

The frequency domain representation of the circuit is shown in Figure . We can analyze this circuit by writing and solving node equations. To simply this process, we can first replace series and parallel impedances by equivalent impedances as shown in Figure . Impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$ in Figure are given by

$$\mathbf{Z}_1 = 10 \| (-j10) = \frac{10(-j10)}{10 - j10} = 5 - j5\,\Omega \text{ and } \mathbf{Z}_2 = 5 + j5\,\Omega$$



…Example 54



Next, consider the dependent source in Figure . We can use Ohm's law to express the controlling current **I** as

$$\mathbf{I} = \frac{\mathbf{V}_s - \mathbf{V}_1}{R_1}$$

Using KVL, we can express the dependent source voltage as

$$10\mathbf{I} = \mathbf{V}_2 - \mathbf{V}_1$$

Apply KCL to the supernode identified in Figure to get

$$\mathbf{I} = \frac{\mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_2}{\mathbf{Z}_3} = \frac{\mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_1 + 10\mathbf{I}}{\mathbf{Z}_3} \quad \Rightarrow \quad (\mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{V}_1 + \mathbf{Z}_2(10 - \mathbf{Z}_3)\mathbf{I} = 0$$

Organizing Eqs. into matrix form, we get

$$\begin{bmatrix} 1 & R_1 \\ \mathbf{Z}_2 + \mathbf{Z}_3 & \mathbf{Z}_2(10 - \mathbf{Z}_3) \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_s \\ 0 \end{bmatrix}$$

Solving these equations, perhaps using MATLAB, gives

$$\mathbf{V}_1 = 4.4721 \angle 63.4^\circ \text{ V and } \mathbf{I} = 0.89443 \angle -26.6^\circ \text{ A}$$

Back in the time domain, the output current is

$$i(t) = 0.89443 \cos(10t - 26.6^\circ) \text{ A}$$

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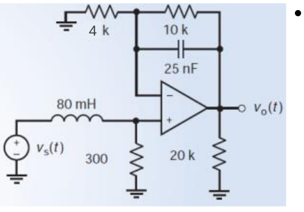
Example 55…

- Input to the circuit is the voltage source voltage $v_a(t) = 125 \cos(500t + 15^o)$ mV.



- Determine the output voltage $i(t)$.

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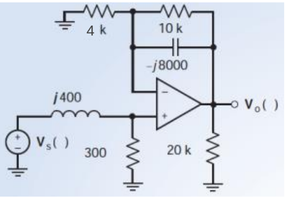
…Example 55…

The impedances of the capacitor and inductor are

$$\mathbf{Z}_C = -j\frac{1}{5000(25 \times 10^{-9})} = -j8000 \ \Omega \text{ and } \mathbf{Z}_L = j5000(80 \times 10^{-3}) = j400 \ \Omega$$

Figure show the circuit represented in the frequency domain using phasors and impedances.



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…Example 55

Applying KCL at the noninverting node of the op amp, we get

$$\frac{\mathbf{V}_s - \mathbf{V}_a}{j400} = \frac{\mathbf{V}_a}{300} + 0 \quad \Rightarrow \quad \mathbf{V}_s = \mathbf{V}_a\left(1 + \frac{j400}{300}\right)$$

Solving for $\mathbf{V}_a$ gives

$$\mathbf{V}_a = \left(\frac{300}{300 + j400}\right)\mathbf{V}_s = \left(0.6 \angle -53.1^\circ\right)\left(0.125 \angle 15^\circ\right) = 0.075 \angle -38.1^\circ \text{ V}$$

Next, apply KCL at the inverting node of the op amp to get

$$\frac{\mathbf{V}_a}{4000} + \frac{\mathbf{V}_a - \mathbf{V}_o}{10,000} + \frac{\mathbf{V}_a - \mathbf{V}_o}{-j8000} = 0$$

Multiplying by 80,000 gives

$$0 = 20\mathbf{V}_a + 8(\mathbf{V}_a - \mathbf{V}_o) + j10(\mathbf{V}_a - \mathbf{V}_o)$$

Solving for $\mathbf{V}_o$ gives

$$\mathbf{V}_o = \frac{28 + j10}{8 + j10}\mathbf{V}_a = \frac{29.73 \angle 19.65^\circ}{12.81 \angle 51.34^\circ}\left(0.075 \angle -38.1^\circ\right) = 0.174 \angle -69.79^\circ$$

In the time domain, the output voltage is

$$v_o(t) = 174 \cos(500t - 69.79^\circ) \text{ mV}$$

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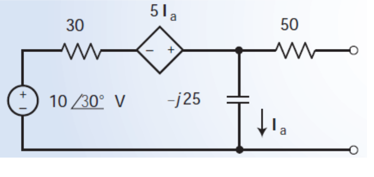
Example 56…

- Find the Norton equivalent circuit of the following ac circuit.



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…Example 56…

In Figure , an open circuit is connected across the terminals of circuit. The voltage across that open circuit is the open-circuit voltage $\mathbf{V}_{oc}$. (Notice that there is no current in the 50-Ω impedance due to the open circuit.) Apply KVL to the left mesh to get

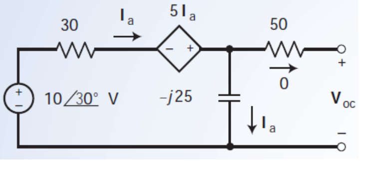
$$30\mathbf{I}_a - 5\mathbf{I}_a + (-j25)\mathbf{I}_a = 10 \angle 30^\circ$$

Solving for $\mathbf{I}_a$, we get

$$\mathbf{I}_a = \frac{10 \angle 30^\circ}{25 - j25} = 0.2828 \angle 75^\circ \text{ A}$$

Apply KVL to the right mesh to get

$$\mathbf{V}_{oc} = -j25\mathbf{I}_a = \left(25 \angle -90^\circ\right)0.2828 \angle 75^\circ = 7.071 \angle -15^\circ \text{ V}$$



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…Example 56…

Next, we determine the short-circuit current using the circuit shown in Figure in Slt.12. In Figure in Slt.12, a short circuit is connected across the terminals of circuit. The current in that open circuit is the short-circuit current $\mathbf{I}_{sc}$. In Figure in Slt.12, the controlling current of the dependent source is related to the mesh currents by

$$\mathbf{I}_a = \mathbf{I}_1 - \mathbf{I}_{sc}$$

Apply KVL to the left mesh to get

$$30\mathbf{I}_1 - 5(\mathbf{I}_1 - \mathbf{I}_{sc}) - j25(\mathbf{I}_1 - \mathbf{I}_{sc}) = 10\underline{/30^\circ}$$

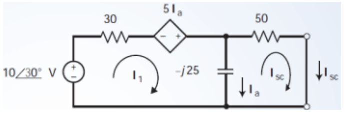
Apply KVL to the left mesh to get

$$50\mathbf{I}_{sc} - (-j25)(\mathbf{I}_1 - \mathbf{I}_{sc}) = 0$$

Organize these equations in matrix form to get

$$\begin{bmatrix} 25-j25 & 5+j25 \\ j25 & 50-j25 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_{sc} \end{bmatrix} = \begin{bmatrix} 10\underline{/30^\circ} \\ 0 \end{bmatrix}$$

Solving using MATLAB gives $\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_{sc} \end{bmatrix} = \begin{bmatrix} 0.2370\underline{/61.4^\circ} \\ 0.1060\underline{/-2^\circ} \end{bmatrix}$



…Example 56

The Thévenin impedance is

$$\mathbf{Z}_t = \frac{\mathbf{V}_{oc}}{\mathbf{I}_{sc}} = \frac{7.071\underline{/-15^\circ}}{0.1060\underline{/-2^\circ}} = 66.71\underline{/-13^\circ}\ \Omega$$

Finally, Figure 10.7.10 shows the Norton equivalent circuit, which consists of a current source in parallel with an impedance. The current source current is the short-circuit voltage $\mathbf{I}_{sc}$. The impedance is the Thévenin impedance $\mathbf{Z}_t$.