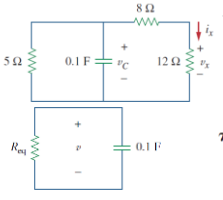
BLM1612 - Circuit Theory

Examples

Example 35



- In the circuit, if $v_C(0)$ =15 V, find v_C, v_x, i_x for $t > 0$.

$$R_{eq} = \frac{20 \times 5}{20 + 5} = 4\ \Omega$$

$$\tau = R_{eq}C = 4(0.1) = 0.4\ \text{s}$$

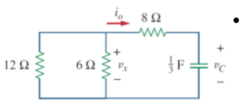
$$v = v(0)e^{-t/\tau} = 15e^{-t/0.4}\ \text{V}$$

$$v_C = v = 15e^{-2.5t}\ \text{V}$$

$$v_x = \frac{12}{12 + 8}v = 0.6(15e^{-2.5t}) = 9e^{-2.5t}\ \text{V}$$

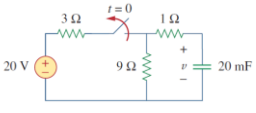
$$i_x = \frac{v_x}{12} = 0.75e^{-2.5t}\ \text{A}$$

Example 36

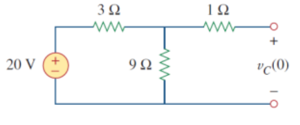


- In the circuit, if $v_C(0)$ =60 V,
  - find v_C, v_x, i_0 for $t \geq 0$.
- Answer: $60e^{-0.25t}$ V, $20e^{-0.25t}$ V, $-5e^{-0.25t}$ A.

Example 37…



- The switch in the circuit has been closed for a long time, and it is opened at $t = 0$.
  - Find $v(t)$ for $t \geq 0$.
  - Calculate the initial energy stored in the capacitor.
- For $t < 0$ the switch is closed; the capacitor is an open circuit to dc.

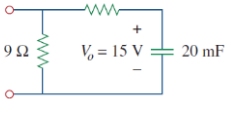


$$v_C(t) = \frac{9}{9 + 3}(20) = 15\ \text{V}$$

$$v_C(0) = V_0 = 15\ \text{V}$$

…Example 37

- For $t > 0$ the switch is open; the voltage source is disconnected.



$$R_{eq} = 1 + 9 = 10\ \Omega$$

$$\tau = R_{eq}C = 10 \times 20 \times 10^{-3} = 0.2\ \text{s}$$

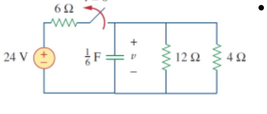
- The voltage across the capacitor for is

$$v(t) = v_C(0)e^{-t/\tau} = 15e^{-t/0.2}\ \text{V} \Rightarrow v(t) = 15e^{-5t}\ \text{V}$$

- The initial energy stored in the capacitor is

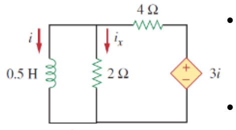
$$w_C(0) = \frac{1}{2}Cv_C^2(0) = \frac{1}{2} \times 20 \times 10^{-3} \times 15^2 = 2.25\ \text{J}$$

Example 38

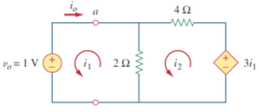


- If the switch in the circuit opens at $t = 0$,
  - find $v(t)$ for $t \geq 0$ and $w_C(0)$.
- Answer: $8e^{-2t}$ V, 5.333 J.

Example 39…



- In the circuit, if $i(0)$ =10 A,
  – find $i(t)$ and v_x, $i_x(t)$ for $t > 0$.
- Method 1:



$$2(i_1 - i_2) + 1 = 0 \Longrightarrow i_1 - i_2 = -\frac{1}{2}$$

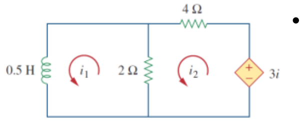
$$6i_2 - 2i_1 - 3i_1 = 0 \Longrightarrow i_2 = \frac{5}{6}i_1$$

$$i_1 = -3\text{ A}, \qquad i_o = -i_1 = 3\text{ A}$$

$$R_{eq} = R_{Th} = \frac{v_o}{i_o} = \frac{1}{3}\,\Omega \qquad \tau = \frac{L}{R_{eq}} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{3}{2}\text{ s}$$

$$i(t) = i(0)e^{-t/\tau} = 10e^{-(2/3)t}\text{ A}, \qquad t > 0$$

…Example 39



- Method 2:

$$\frac{1}{2}\frac{di_1}{dt} + 2(i_1 - i_2) = 0$$

$$\frac{di_1}{dt} + 4i_1 - 4i_2 = 0$$

$$6i_2 - 2i_1 - 3i_1 = 0 \Longrightarrow i_2 = \frac{5}{6}i_1$$

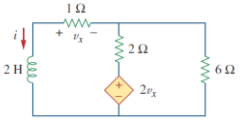
$$\frac{di_1}{dt} + \frac{2}{3}i_1 = 0$$

$$\frac{di_1}{i_1} = -\frac{2}{3}dt \qquad i_1 = i \qquad \frac{di}{i} = -\frac{2}{3}dt \Longrightarrow \int_{i(0)}^{i(t)} \frac{di}{i} = -\frac{2}{3}\int_0^t dt$$

$$\ln i\Big|_{i(0)}^{i(t)} = -\frac{2}{3}t\Big|_0^t \Longrightarrow \ln\frac{i(t)}{i(0)} = -\frac{2}{3}t$$

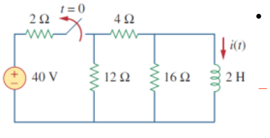
$$i(t) = i(0)e^{-(2/3)t} = 10e^{-(2/3)t}\text{ A}, \qquad t > 0$$

Example 40

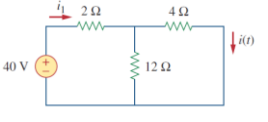


- In the circuit, if $i(0)$ = 12 A,
  – find i and v_x.
- Answer: $12e^{-2t}$ A, $-12e^{-2t}$ V, $t > 0$.

Example 41…



- The switch in the circuit has been closed for a long time, and it is opened at t = 0.
  – Find $i(t)$ for $t > 0$.
- For $t < 0$ the switch is closed; the inductor is short circuit in dc.



$$\frac{4 \times 12}{4 + 12} = 3\,\Omega \qquad i_1 = \frac{40}{2 + 3} = 8\text{ A}$$

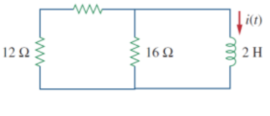
$$i(t) = \frac{12}{12 + 4}i_1 = 6\text{ A}, \qquad t < 0$$

…Example 41

- Since the current through an inductor cannot change instantaneously,

$$i(0) = i(0^-) = 6\text{ A}$$

- For $t > 0$ the switch is open; the voltage source is disconnected.

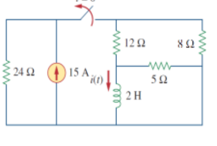


$$R_{eq} = (12 + 4) \parallel 16 = 8\,\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{2}{8} = \frac{1}{4}\text{ s}$$

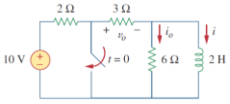
$$i(t) = i(0)e^{-t/\tau} = 6e^{-4t}\text{ A}$$

Example 42

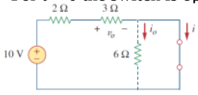


- For the circuit, find $i(t)$ for $t > 0$.
- Answer: $5e^{-2t}$ A, $t > 0$.

Example 43…



- The switch in the circuit has been open for a long time, and it is closed at $t = 0$.
  - Find i_0, v_0, and i for all time.
- For $t < 0$ the switch is open; the inductor is short circuit in dc.

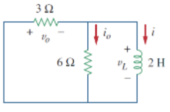


$$i_o = 0 \qquad i(t) = \frac{10}{2+3} = 2 \text{ A}, \quad t < 0$$

$$v_o(t) = 3i(t) = 6 \text{ V}, \quad t < 0$$

$$i(0) = 2$$

- For $t > 0$ the switch is closed; voltage source is short-circuited



$$R_{Th} = 3 \parallel 6 = 2\ \Omega \qquad \tau = \frac{L}{R_{Th}} = 1 \text{ s}$$

$$i(t) = i(0)e^{-t/\tau} = 2e^{-t} \text{ A}, \quad t > 0$$

…Example 43

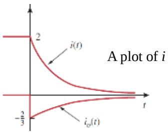
- Since the inductor is in parallel with the 6 Ω and 3 Ω resistors,

$$v_o(t) = -v_L = -L\frac{di}{dt} = -2(-2e^{-t}) = 4e^{-t} \text{ V}, \quad t > 0$$

$$i_o(t) = \frac{v_L}{6} = -\frac{2}{3}e^{-t} \text{ A}, \quad t > 0$$

- For all time,

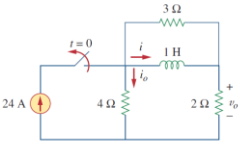
$$i(t) = \begin{cases} 2 \text{ A}, & t < 0 \\ 2e^{-t} \text{ A}, & t \ge 0 \end{cases}$$



A plot of i and i_o

$$i_o(t) = \begin{cases} 0 \text{ A}, & t < 0 \\ -\frac{2}{3}e^{-t} \text{ A}, & t > 0 \end{cases}, \qquad v_o(t) = \begin{cases} 6 \text{ V}, & t < 0 \\ 4e^{-t} \text{ V}, & t > 0 \end{cases}$$

Example 44

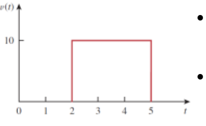


- Determine i, i_o, and v_o for all t in the circuit shown.
- Assume that the switch was closed for a long time.
- Answer:

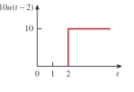
$$i = \begin{cases} 16 \text{ A}, & t < 0 \\ 16e^{-2t} \text{ A}, & t \ge 0 \end{cases}, \qquad i_o = \begin{cases} 8 \text{ A}, & t < 0 \\ -5.333e^{-2t} \text{ A}, & t > 0 \end{cases},$$

$$v_o = \begin{cases} 32 \text{ V}, & t < 0 \\ 10.667e^{-2t} \text{ V}, & t > 0 \end{cases}$$

Example 45



- Express the voltage pulse (gate function) in terms of the unit step.
- Calculate its derivative and sketch it.



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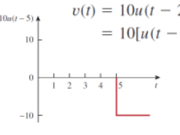
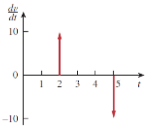
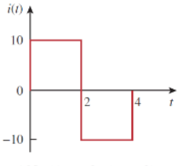


$$v(t) = 10u(t-2) - 10u(t-5)$$
$$= 10[u(t-2) - u(t-5)]$$



$$\frac{dv}{dt} = 10[\delta(t-2) - \delta(t-5)]$$

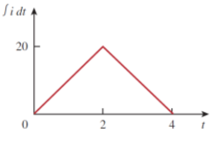
Example 46



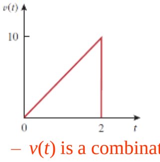
- Express the current pulse in the figure in terms of the unit step.
- Find its integral and sketch it.
- Answer:

$$10[u(t) - 2u(t-2) + u(t-4)],$$

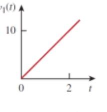
$$10[r(t) - 2r(t-2) + r(t-4)]$$



Example 47…



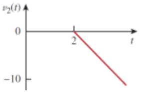
- Express the sawtooth function shown in the figüre in terms of singularity functions.
- Solution 1:
  - $v(t)$ is a combination of singularity functions



$$v_1(t) = 5r(t)$$

+

$$v_2(t) = -5r(t-2)$$



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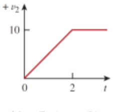
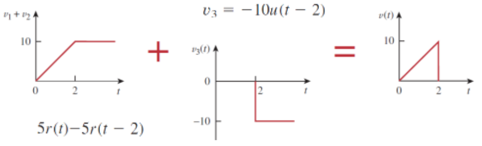


$$5r(t) - 5r(t-2)$$

…Example 47…

$v_3 = -10u(t-2)$



$5r(t) - 5r(t-2)$

$$v(t) = v_1(t) + v_2(t) + v_3(t)$$

$$v(t) = 5r(t) - 5r(t-2) - 10u(t-2)$$

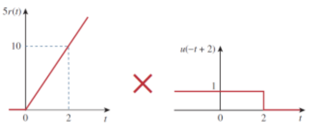
…Example 47…

- Solution 2:
  - $v(t)$ is a multiplication of two functions:
    - a ramp function and a gate function.

$$\begin{aligned} v(t) &= 5t[u(t) - u(t-2)] \\ &= 5tu(t) - 5tu(t-2) \\ &= 5r(t) - 5(t-2+2)u(t-2) \\ &= 5r(t) - 5(t-2)u(t-2) - 10u(t-2) \\ &= 5r(t) - 5r(t-2) - 10u(t-2) \end{aligned}$$

…Example 47

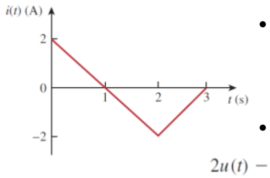
- Solution 3:
  - $v(t)$ is a multiplication of two functions:
    - a ramp function and a unit step function.



$$v(t) = 5r(t)u(-t+2)$$

$$v(t) = 5r(t)[1 - u(t-2)]$$

Example 48



- Express $i(t)$ in terms of singularity functions.
- Answer:

$2u(t) - 2r(t) + 4r(t-2) - 2r(t-3)$ A

Example 49

- Express the signal $g(t)$ in terms of step and ramp functions.

$$g(t) = \begin{cases} 3, & t < 0 \\ -2, & 0 < t < 1 \\ 2t-4, & t > 1 \end{cases}$$

$$\begin{aligned} g(t) &= 3u(-t) - 2[u(t) - u(t-1)] + (2t-4)u(t-1) \\ &= 3u(-t) - 2u(t) + (2t-4+2)u(t-1) \\ &= 3u(-t) - 2u(t) + 2(t-1)u(t-1) \\ &= 3u(-t) - 2u(t) + 2r(t-1) \end{aligned}$$

$$g(t) = 3[1 - u(t)] - 2u(t) + 2r(t-1) = 3 - 5u(t) + 2r(t-1)$$

Example 50

$$h(t) = \begin{cases} 0, & t < 0 \\ -4, & 0 < t < 2 \\ 3t-8, & 2 < t < 6 \\ 0, & t > 6 \end{cases}$$

- Express $h(t)$ in terms of the singularity functions.
- Answer:

$$-4u(t) + 2u(t-2) + 3r(t-2) - 10u(t-6) - 3r(t-6)$$

Example 51

- Evaluate the following integrals involving the impulse function: $\int_0^{10} (t^2 + 4t - 2)\delta(t - 2)dt$, $\int_{-\infty}^{\infty} [\delta(t-1)e^{-t}\cos t + \delta(t+1)e^{-t}\sin t]dt$

$$\int_0^{10} (t^2 + 4t - 2)\delta(t - 2)dt = (t^2 + 4t - 2)|_{t=2} = 4 + 8 - 2 = 1($$

$$\int_{-\infty}^{\infty} [\delta(t-1)e^{-t}\cos t + \delta(t+1)e^{-t}\sin t]dt$$
$$= e^{-t}\cos t|_{t=1} + e^{-t}\sin t|_{t=-1}$$
$$= e^{-1}\cos 1 + e^{1}\sin(-1) = 0.1988 - 2.2873 = -2.0885$$

Example 52

- Evaluate the following integrals:

$$\int_{-\infty}^{\infty} (t^3 + 5t^2 + 10)\delta(t+3)dt, \qquad \int_0^{10} \delta(t - \pi)\cos 3t\, dt$$

- Answer: 28, -1