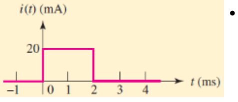
BLM1612 - Circuit Theory

Examples

Example 18



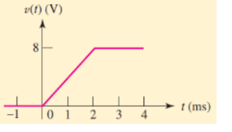
- Find the capacitor voltage that is associated with the current shown graphically in the Figure.
  - The value of the capacitance is 5 µF.

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t')\,dt' + v(t_0)$$

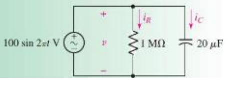
$$v(t) = \frac{1}{5\times 10^{-6}}\int_{0}^{t} 20\times 10^{-3}\,dt' + v(0)$$

Since $v(0) = 0$,

$$v(t) = 4000t \qquad 0 \le t \le 2\text{ ms}$$



Example 18…

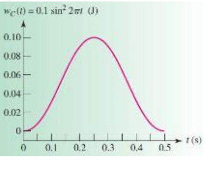


- Find the maximum energy stored in the capacitor and the energy dissipated in the resistor over the interval 0 < t < 0.5 s.

$$w_C = \frac{1}{2}C\cdot v^2$$

$$w_C = \frac{1}{2}(20\ \mu\text{F})\cdot\{100\sin(2\pi t)\ \text{V}\}^2$$

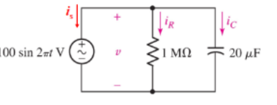
$$= 100\sin^2(2\pi t)\ \text{mJ}$$



…Example 18…

$$i_C = C\cdot\frac{dv}{dt} = (20\ \mu)\cdot\frac{d}{dt}100\sin(2\pi t)$$

$$= (2\ \text{m})\cdot 2\pi\cdot\cos(2\pi t) = 4\pi\cdot\cos(2\pi t)\ \text{mA}$$



$$i_R = \frac{v}{R} = \frac{100\sin(2\pi t)\ \text{V}}{10^6\ \Omega} = 0.1\sin(2\pi t)\ \text{mA}$$

$$i_s + i_R + i_C = 0$$

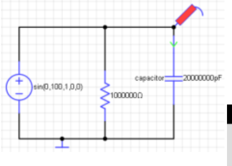
$$i_s = -0.1\sin(2\pi t) - 4\pi\cdot\cos(2\pi t)\ \text{mA}$$

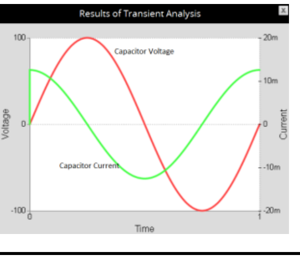
$$p_R = i_R^2 R = (10^{-8})(10^6)\sin^2 2\pi t$$

so that the energy dissipated in the resistor between 0 and 0.5 s is

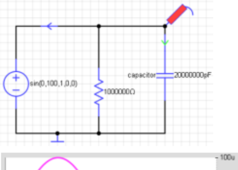
$$w_R = \int_0^{0.5} p_R\,dt = \int_0^{0.5} 10^{-2}\sin^2 2\pi t\,dt \qquad \text{J}$$

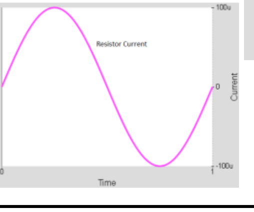
…Example 18…

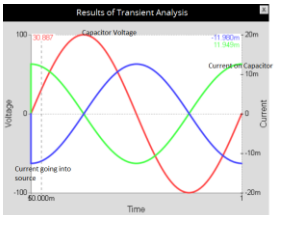




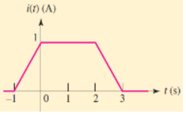
…Example 18







Example 19



- Given the waveform of the current in a 3 H inductor as shown in the Figure, determine the inductor voltage and sketch it.

$$v = 3\frac{di}{dt}$$

t < -1 sec i = 0 ampere

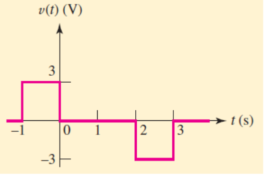
0 sec < t < 2 sec i = 1 ampere

t > 3 sec i = 0 ampere

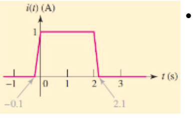
the current is constant, $v = 0$

-1 sec <= t <= 0 sec di/dt = 1 A/s
v = 3 volt

2 sec <= t <= 3 sec di/dt = -1 A/s
v = -3 volt



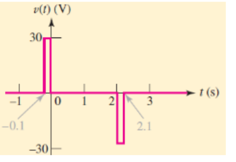
Example 20



- Given the waveform of the current in a 3 H inductor as shown in the Figure, determine the inductor voltage and sketch it.

$$v = 3\frac{di}{dt}$$

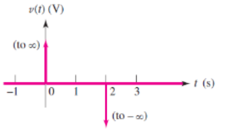
- The intervals for the rise and fall have decreased to 0.1 s.
  - Thus, the magnitude of each derivative will be 10 times larger.



- It is interesting to note that the area under each voltage pulse is 3 V× s.

Example 21

- A further decrease in the rise and fall times of the current waveform will produce a proportionally larger voltage magnitude, but only within the interval in which the current is increasing or decreasing.
  - An abrupt change in the current will cause the infinite voltage "spikes" (each having an area of 3 V×s)



- This is useful in the **ignition system of some automobiles**, where the current through the spark coil is interrupted by the distributor and the arc appears across the spark plug.

Example 22

- The voltage across a 2 H inductor is known to be 6cos5t V.
  - Determine the resulting inductor current if $i(t = -\pi/2) = 1$A.

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)\cdot d\tau + i(t_0)$$

$$i(t) = \frac{1}{2}\int_{t_0}^{t} 6\cos 5t'\, dt' + i(t_0)$$

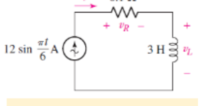
$$i(t) = \frac{1}{2}\left(\frac{6}{5}\right)\sin 5t - \frac{1}{2}\left(\frac{6}{5}\right)\sin 5t_0 + i(t_0)$$
$$= 0.6\sin 5t - 0.6\sin 5t_0 + i(t_0)$$

- If we accept $t_0 = -\pi/2$

$$i(t) = 0.6\sin 5t - 0.6\sin(-2.5\pi) + 1$$

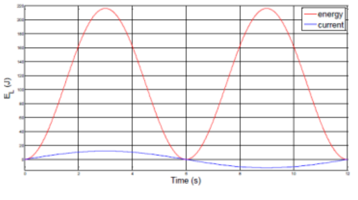
$$i(t) = 0.6\sin 5t + 1.6$$

Example 23…

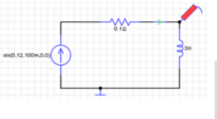


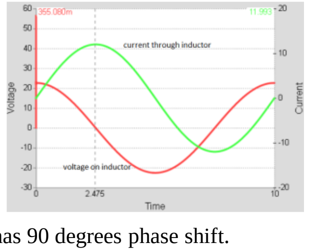
$$w_L = \frac{1}{2}Li^2 = 216\sin^2\frac{\pi t}{6} \quad \text{J}$$

- Find the maximum energy stored in the inductor and calculate how much energy is dissipated in the resistor in the time during which the energy is being stored in, and then recovered from, the inductor.



…Example 23…





- Current and voltage has 90 degrees phase shift.
  - Voltage is leading the current.

…Example 23

- The power dissipated in the resistor is easily found as

$$p_R = i^2 R = 14.4 \sin^2 \frac{\pi t}{6} \quad \text{W}$$

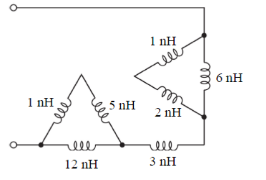
- The energy converted into heat in the resistor within this 6 s interval is

$$w_R = \int_0^6 p_R\, dt = \int_0^6 14.4 \sin^2 \frac{\pi}{6} t\, dt$$

$$w_R = \int_0^6 14.4 \left(\frac{1}{2}\right)\left(1 - \cos \frac{\pi}{3} t\right) dt = 43.2 \text{ J}$$

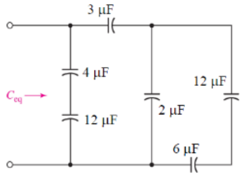
Example 24

- Determine the equivalent inductance (L_{eq}) at the open-circuit terminals.

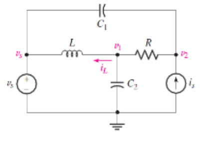


Example 25

- Determine the equivalent capacitance (C_{eq}) at the open-circuit terminals.



Example 26…



- Write appropriate nodal equations for the circuit.
- KCL at v_1 node

$$\frac{1}{L}\int_{t_0}^{t} (v_1 - v_s)\, dt' + i_L(t_0) + \frac{v_1 - v_2}{R} + C_2 \frac{dv_1}{dt} = 0$$

- KCL at v_2 node

$$C_1 \frac{d(v_2 - v_s)}{dt} + \frac{v_2 - v_1}{R} - i_s = 0$$

- When we rewrite these equations
  - obtain linear integro-differential equations

$$\frac{v_1}{R} + C_2 \frac{dv_1}{dt} + \frac{1}{L}\int_{t_0}^{t} v_1\, dt' - \frac{v_2}{R} = \frac{1}{L}\int_{t_0}^{t} v_s\, dt' - i_L(t_0)$$

$$-\frac{v_1}{R} + \frac{v_2}{R} + C_1 \frac{dv_2}{dt} = C_1 \frac{dv_s}{dt} + i_s$$

…Example 26

- We will not attempt the solution of integro-differential equations here.
  - It is worthwhile pointing out, however, that when the voltage forcing functions are sinusoidal functions of time, it will be possible to define a voltage-current ratio (called impedance) or a current-voltage ratio (called admittance) for each of the three passive elements.
- The factors operating on the two node voltages in the preceding equations will then become simple multiplying factors, and the equations will be linear algebraic equations once again.
- • These we may solve by determinants or a simple elimination of variables as before.