

BLM1612 - Circuit Theory

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Capacitors
Inductors

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Energy Storage Devices

Capacitors and Inductors

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Objective of Lecture

- Describe
 - the construction of a capacitor
 - how charge is stored.
 - Introduce several types of capacitors
- The electrical properties of a capacitor
 - Relationship between charge, voltage, and capacitance; power; and energy
 - Equivalent capacitance when a set of capacitors are in series and in parallel
- Describe
 - The construction of an inductor
 - How energy is stored in an inductor
 - The electrical properties of an inductor
 - Relationship between voltage, current, and inductance; power; and energy
 - Equivalent inductance when a set of inductors are in series and in parallel

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Capacitors

Energy Storage Devices

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Capacitors

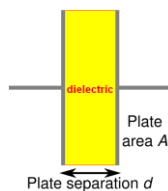
- Composed of two conductive plates separated by an insulator (or dielectric).
 - Commonly illustrated as two parallel metal plates separated by a distance, d .

$$C = \epsilon A/d$$

where $\epsilon = \epsilon_r \epsilon_0$

ϵ_r is the relative dielectric constant

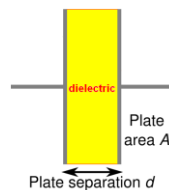
ϵ_0 is the vacuum permittivity



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Effect of Dimensions

- Capacitance increases with
 - increasing surface area of the plates,
 - decreasing spacing between plates, and
 - increasing the relative dielectric constant of the insulator between the two plates.



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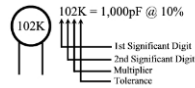
Types of Capacitors

- Fixed Capacitors
 - Nonpolarized
 - May be connected into circuit with either terminal of capacitor connected to the high voltage side of the circuit.
 - Insulator: Paper, Mica, Ceramic, Polymer
 - Electrolytic
 - The negative terminal must always be at a lower voltage than the positive terminal
 - Plates or Electrodes: Aluminum, Tantalum

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Nonpolarized

- Difficult to make nonpolarized capacitors that store a large amount of charge or operate at high voltages.
 - Tolerance on capacitance values is very large
 - +50%/-25% is not unusual



PSpice Symbol



http://www.marvac.com/fun/ceramic_capacitor_codes.aspx

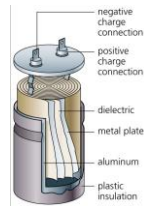
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Electrolytic

PSpice Symbols



Fabrication

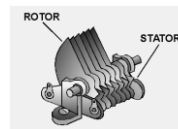


<http://www.digitivty.com/articles/2008/11/choosing-the-right-capacitor.html>

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Variable Capacitors

- Cross-sectional area is changed as one set of plates are rotated with respect to the other.



PSpice Symbol

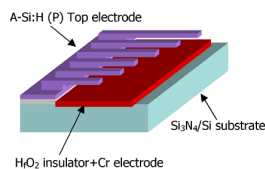


<http://www.tpub.com/meets/book2/3f.htm>

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MEMS Capacitor

- MEMS (Microelectromechanical system)
 - Can be a variable capacitor by changing the distance between electrodes.
 - Use in sensing applications as well as in RF electronics.



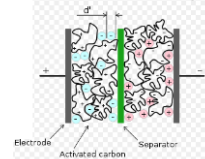
http://www.silvaco.com/tech_lib_TCAD/simulationstandard/2005/aug/a3/a3.html

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Electric Double Layer Capacitor

- Also known as a supercapacitor or ultracapacitor
 - Used in high voltage/high current applications.
 - Energy storage for alternate energy systems.

Electrochemical double-layer



http://en.wikipedia.org/wiki/File:Supercapacitor_diagram.svg

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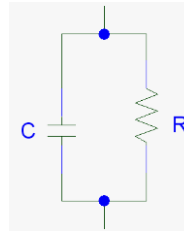
Electrical Properties of a Capacitor

- Acts like an open circuit at steady state when connected to a d.c. voltage or current source.
- Voltage on a capacitor must be continuous
 - There are no abrupt changes to the voltage
- An ideal capacitor does not dissipate energy, it takes power when storing energy and returns it when discharging.

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Properties of a Real Capacitor

- A real capacitor does dissipate energy due to leakage of charge through its insulator.

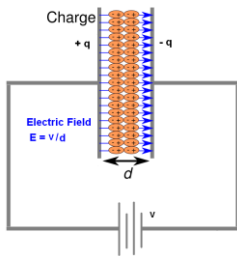


– This is modeled by putting a resistor in parallel with an ideal capacitor.

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Energy Storage

- Charge is stored on the plates of the capacitor.



Equation:

$$Q = CV$$

Units:

$$\text{Coulomb} = \text{Farad} \times \text{Voltage}$$

$$C = F \cdot V$$

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Adding Charge to Capacitor

- The ability to add charge to a capacitor depends on:
 - the amount of charge already on the plates of the capacitor
- and
 - the force (voltage) driving the charge towards the plates (i.e., current)

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Example 01

- A 6 V source is required to store 24 μC of charge on a certain capacitor.
 - What is the capacitance of the capacitor?

$$C = \frac{Q}{V} = \frac{24 \times 10^{-6} \text{ C}}{6 \text{ V}} = 4 \times 10^{-6} \text{ F} = 4 \mu\text{F}$$
 - How much charge is stored on the capacitor when a 9 V source is connected?

$$Q = C \times V = 4 \mu\text{F} \times 9 \text{ V} = 36 \mu\text{C}$$
 - What is the voltage across the capacitor when 16 μC is stored on it?

$$V = \frac{Q}{C} = \frac{16 \mu\text{C}}{4 \mu\text{F}} = 4 \text{ V}$$

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Charging a Capacitor

- At first, it is easy to store charge in the capacitor.
- As more charge is stored on the plates of the capacitor, it becomes increasingly difficult to place additional charge on the plates.
 - Coulombic repulsion from the charge already on the plates creates an opposing force to limit the addition of more charge on the plates.
 - Voltage across a capacitor increases rapidly as charge is moved onto the plates when the initial amount of charge on the capacitor is small.
 - Voltage across the capacitor increases more slowly as it becomes difficult to add extra charge to the plates.

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Discharging a Capacitor

- At first, it is easy to remove charge in the capacitor.
 - Coulombic repulsion from the charge already on the plates creates a force that pushes some of the charge out of the capacitor once the force (voltage) that placed the charge in the capacitor is removed (or decreased).
- As more charge is removed from the plates of the capacitor, it becomes increasingly difficult to get rid of the small amount of charge remaining on the plates.
 - Coulombic repulsion decreases as the charge spreads out on the plates. As the amount of charge decreases, the force needed to drive the charge off of the plates decreases.
 - Voltage across a capacitor decreases rapidly as charge is removed from the plates when the initial amount of charge on the capacitor is small.
 - Voltage across the capacitor decreases more slowly as it becomes difficult to force the remaining charge out of the capacitor.

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Current-Voltage Relationships

$$i_c = \frac{dq}{dt}$$

$$q = Cv_C$$

$$i_c = C \frac{dv_C}{dt}$$

$$v_C = \frac{1}{C} \int_{t_0}^{t_1} i_c dt$$

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Power and Energy

$$p_C = i_C v_C$$

$$w_C = \frac{1}{2} C v_C^2$$

$$p_C = C v_C \frac{dv_C}{dt}$$

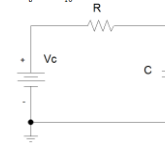
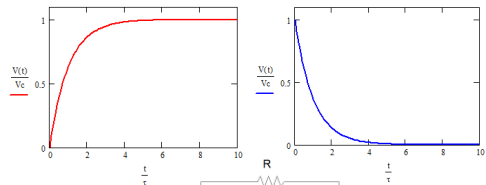
$$w_C = \frac{q^2}{2C}$$

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Capacitor Voltage vs. Time

d.c. voltage, V_C , is applied at $t = 0$ s

d.c. voltage, V_C , is removed at $t = 0$ s



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Time constant, τ

- The rate at which charge can be added to or removed from the plates of a capacitor as a function of time can be fit to an exponential function.

Charging

Discharging

$$V(t) = V_C (1 - e^{-t/\tau})$$

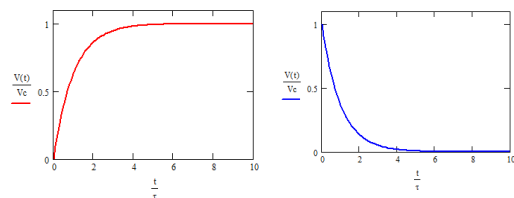
$$V(t) = V_C e^{-t/\tau}$$

$$\tau = RC$$

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Transition to steady state

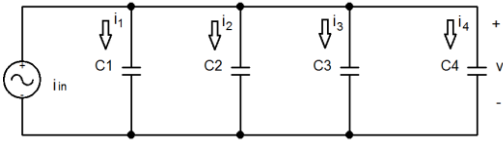
- We approximate that the exponential function reaches its final value when the charging or discharging time is equal to 5τ .



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Equivalent Capacitance

- Capacitors in parallel



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C_{eq} for Capacitors in Parallel

$$i_m = i_1 + i_2 + i_3 + i_4$$

$$i_1 = C_1 \frac{dv}{dt} \quad i_2 = C_2 \frac{dv}{dt}$$

$$i_3 = C_3 \frac{dv}{dt} \quad i_4 = C_4 \frac{dv}{dt}$$

$$i_m = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + C_4 \frac{dv}{dt}$$

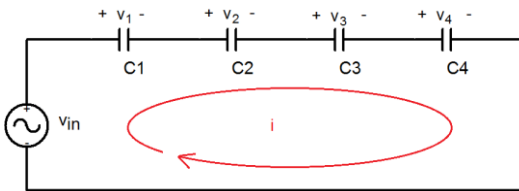
$$i_m = C_{eq} \frac{dv}{dt}$$

$$C_{eq} = C_1 + C_2 + C_3 + C_4$$

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Equivalent Capacitance

- Capacitors in series



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C_{eq} for Capacitors in Series

$$v_m = v_1 + v_2 + v_3 + v_4$$

$$v_1 = \frac{1}{C_1} \int_{t_0}^{t_1} i dt \quad v_2 = \frac{1}{C_2} \int_{t_0}^{t_1} i dt$$

$$v_3 = \frac{1}{C_3} \int_{t_0}^{t_1} i dt \quad v_4 = \frac{1}{C_4} \int_{t_0}^{t_1} i dt$$

$$v_m = \frac{1}{C_1} \int_{t_0}^{t_1} i dt + \frac{1}{C_2} \int_{t_0}^{t_1} i dt + \frac{1}{C_3} \int_{t_0}^{t_1} i dt + \frac{1}{C_4} \int_{t_0}^{t_1} i dt$$

$$v_m = \frac{1}{C_{eq}} \int_{t_0}^{t_1} i dt$$

$$C_{eq} = \left[\left(\frac{1}{C_1} \right) + \left(\frac{1}{C_2} \right) + \left(\frac{1}{C_3} \right) + \left(\frac{1}{C_4} \right) \right]^{-1}$$

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General Equations for C_{eq}

Parallel Combination

- If P capacitors are in parallel, then

$$C_{eq} = \sum_{p=1}^P C_p$$

Series Combination

- If S capacitors are in series, then:

$$C_{eq} = \left[\sum_{s=1}^S \frac{1}{C_s} \right]^{-1}$$

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Summary

- Capacitors are energy storage devices.
- An ideal capacitor act like an open circuits when a DC voltage or current has been applied for at least 5τ .
- The voltage across a capacitor must be a continuous function; the current flowing across a capacitor can be discontinuous.
- The equation for equivalent capacitance for

capacitors in parallel

$$C_{eq} = \sum_{p=1}^P C_p$$

capacitors in series

$$C_{eq} = \left[\sum_{s=1}^S \frac{1}{C_s} \right]^{-1}$$

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Inductors

Energy Storage Devices

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Inductors

- Generally - coil of conducting wire



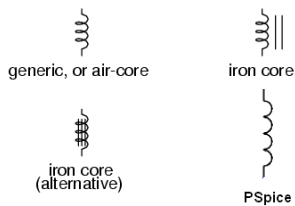
- Usually wrapped around a solid core.
- If no core is used, then the inductor is said to have an ‘air core’.

<http://bzupages.com/f231/energy-stored-inductor-uzma-noreen-group6-part2-3464/>

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Symbols

Inductor symbols



http://www.allaboutcircuits.com/vol_1/chpt_15/1.html

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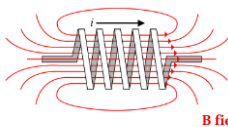
Alternative Names for Inductors

- Reactor
 - inductor in a power grid
- Choke
 - designed to block a particular frequency while allowing currents at lower frequencies or d.c. currents through
 - Commonly used in RF (radio frequency) circuitry
- Coil
 - often coated with varnish and/or wrapped with insulating tape to provide additional insulation and secure them in place
 - A winding is a coil with taps (terminals).
- Solenoid
 - a three dimensional coil.
 - Also used to denote an electromagnet where the magnetic field is generated by current flowing through a toroidal inductor.

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Energy Storage

- The flow of current through an inductor creates a magnetic field (right hand rule).



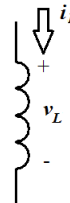
http://en.wikibooks.org/wiki/Circuit_Theory/Mutual_Inductance

- If the current flowing through the inductor drops, the magnetic field will also decrease and energy is released through the generation of a current.

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Sign Convention

- The sign convention used with an inductor is the same as for a power dissipating device.



- When current flows into the positive side of the voltage across the inductor, it is positive and the inductor is dissipating power.
- When the inductor releases energy back into the circuit, the sign of the current will be negative.

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Current and Voltage Relationships

- L, inductance, has the units of Henries (H)
 $1 \text{ H} = 1 \text{ V}\cdot\text{s}/\text{A}$

$$v_L = L \frac{di}{dt}$$

$$i_L = \frac{1}{L} \int_{t_0}^{t_1} v_L dt$$

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Power and Energy

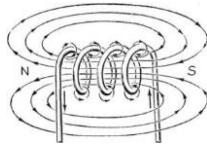
$$p_L = v_L i_L = L i_L \int_{t_0}^{t_1} i_L dt$$

$$w = \int_{t_0}^{t_1} L \frac{di_L}{dt} i_L dt = L \int_{t_0}^{t_1} i_L di_L$$

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Inductors

- Stores energy in an magnetic field created by the electric current flowing through it.
 - Inductor opposes change in current flowing through it.
 - Current through an inductor is continuous; voltage can be discontinuous.



<http://www.rfcafe.com/references/electrical/Electricity%20-%20Basic%20Navy%20Training%20Courses/electricity%20-%20basic%20navy%20training%20courses%20-%20chapter%20.htm>

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Calculations of L

- For a solenoid (toroidal inductor)

$$L = \frac{N^2 \mu A}{\ell} = \frac{N^2 \mu_r \mu_o A}{\ell}$$

- N is the number of turns of wire
- A is the cross-sectional area of the toroid in m².
- μ_r is the relative permeability of the core material
- μ_o is the vacuum permeability ($4\pi \times 10^{-7} \text{ H/m}$)
- ℓ is the length of the wire used to wrap the toroid in meters

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Wire

- Unfortunately, even bare wire has inductance.



$$L = \left[\ln \left(4 \frac{\ell}{d} \right) - 1 \right] (2 \times 10^{-7}) H$$

- d is the diameter of the wire in meters.

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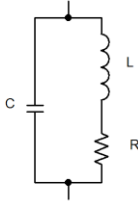
Properties of an Inductor

- Acts like an short circuit at steady state when connected to a d.c. voltage or current source.
- Current through an inductor must be continuous
 - There are no abrupt changes to the current, but there can be abrupt changes in the voltage across an inductor.
- An ideal inductor does not dissipate energy, it takes power from the circuit when storing energy and returns it when discharging.

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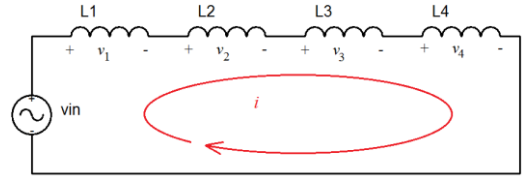
Properties of a Real Inductor

- Real inductors do dissipate energy due to resistive losses in the length of wire and capacitive coupling between turns of the wire.



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Inductors in Series



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L_{eq} for Inductors in Series

$$v_{in} = v_1 + v_2 + v_3 + v_4$$

$$v_1 = L_1 \frac{di}{dt} \quad v_2 = L_2 \frac{di}{dt}$$

$$v_3 = L_3 \frac{di}{dt} \quad v_4 = L_4 \frac{di}{dt}$$

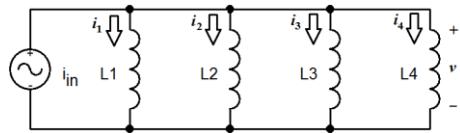
$$v_{in} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + L_4 \frac{di}{dt}$$

$$v_{in} = L_{eq} \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3 + L_4$$

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Inductors in Parallel



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L_{eq} for Inductors in Parallel

$$i_{in} = i_1 + i_2 + i_3 + i_4$$

$$i_1 = \frac{1}{L_1} \int_{t_0}^{t_1} v dt \quad i_2 = \frac{1}{L_2} \int_{t_0}^{t_1} v dt$$

$$i_3 = \frac{1}{L_3} \int_{t_0}^{t_1} v dt \quad i_4 = \frac{1}{L_4} \int_{t_0}^{t_1} v dt$$

$$i_{in} = \frac{1}{L_1} \int_{t_0}^{t_1} v dt + \frac{1}{L_2} \int_{t_0}^{t_1} v dt + \frac{1}{L_3} \int_{t_0}^{t_1} v dt + \frac{1}{L_4} \int_{t_0}^{t_1} v dt$$

$$i_{in} = \frac{1}{L_{eq}} \int_{t_0}^{t_1} v dt$$

$$L_{eq} = \left[(1/L_1) + (1/L_2) + (1/L_3) + (1/L_4) \right]^{-1}$$

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General Equations for L_{eq}

Series Combination

- If S inductors are in series, then

$$L_{eq} = \sum_{s=1}^S L_s$$

Parallel Combination

- If P inductors are in parallel, then:

$$L_{eq} = \left[\sum_{p=1}^P \frac{1}{L_p} \right]^{-1}$$

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Summary

- Inductors are energy storage devices.
- An ideal inductor act like a short circuit at steady state when a DC voltage or current has been applied.
- The current through an inductor must be a continuous function; the voltage across an inductor can be discontinuous.
- The equation for equivalent inductance for

inductors in series

$$L_{eq} = \sum_{s=1}^s L_s$$

inductors in parallel

$$L_{eq} = \left[\sum_{p=1}^p \frac{1}{L_p} \right]^{-1}$$

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