Advanced Digital Signal Processing

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An example:

Processing Complex Quadrature Signals

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Quadrature Signals

- Quadrature signals are based on the notion of complex numbers
- used in many digital signal processing applications such as
  - Communication
  - Radar
  - Sonar
  - Ultrasound
  - MR imaging
  - Direction finding schemes
  - Antenna beamforming applications
  - Single sideband modulators

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- A quadrature signal is a two-dimensional signal whose value at any given time can be specified by a single complex number
  - Such as **a(t)+jb(t)**
- Quadrature signal processing is used in many fields of science and engineering
  - Processing of complex quadrature signal provides additional processing power by enabling to measure amplitude and phase of a signal simultaneously

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- http://www.dspguru.com/dsp/tutorials/quadrature-signals

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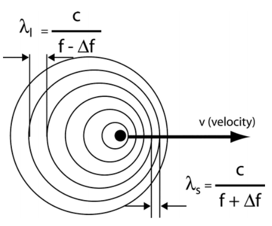
Doppler ultrasound



- As an object emitting sound moves at a velocity *v*,
- the wavelength of the sound in the forward direction is compressed (λ_s) and
- the wavelength of the sound in the receding direction is elongated (λ_l).
- Since frequency (*f*) is inversely related to wavelength, the compression increases the perceived frequency and the elongation decreases the perceived frequency.
- *c* = sound speed.

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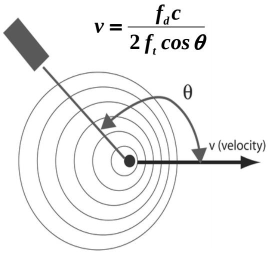
Doppler ultrasound

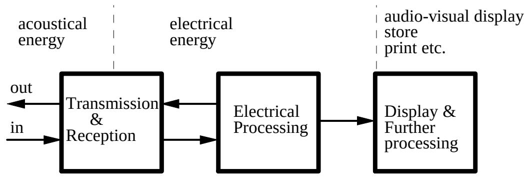
$$f_d = f_t - f_r = \frac{2vf_t \cos\theta}{c}$$

$$v = \frac{f_d c}{2 f_t \cos\theta}$$

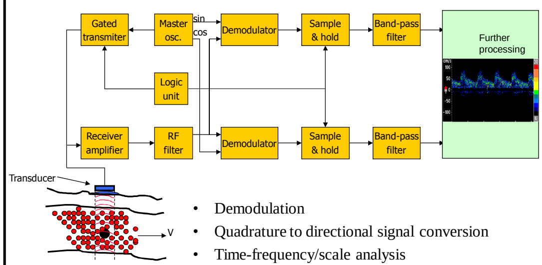


f_t is transmitted frequency
f_r is received frequency
v is the velocity of the target,
θ is the angle between the ultrasound beam and the direction of the target's motion, and
c is the velocity of sound in the medium

A general Doppler ultrasound signal measurement system

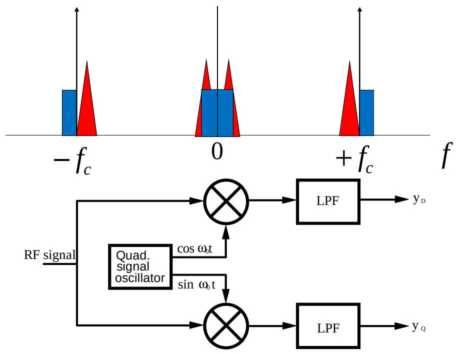


Processing of Doppler Ultrasound Signals



- Demodulation
- Quadrature to directional signal conversion
- Time-frequency/scale analysis
- Data visualization
- Detection and estimation
- Derivation of diagnostic information

Quadrature phase detection



General Definition of Quadrature Doppler Signal

- A general definition of a discrete quadrature Doppler signal equation can be given by

$$\begin{cases} D(n) = \pm s_f(n) \pm H[s_r(n)] \\ Q(n) = \pm H[s_f(n)] \pm s_r(n) \end{cases}$$

- ***D*(*n*)** and ***Q*(*n*),** each containing information concerning forward channel and reverse channel signals (**$s_f(n)$** and **$s_r(n)$**) and their Hilbert transforms **$H[s_f(n)]$** and **$H[s_r(n)]$**), are real signals.

Properties of Complex Quadrature Doppler US signals

$$s(t) = D + jQ, \qquad D = H[Q]$$

$$S(\omega) = S_R^+(\omega) + S_R^-(\omega) + j(S_I^+(\omega) + S_I^-(\omega))$$

$$S_f(\omega) = \begin{cases} 2S(\omega) & if \quad \omega \geq 0 \\ 0 & if \quad \omega < 0 \end{cases}, \; S_r(\omega) = \begin{cases} 0 & if \quad \omega \geq 0 \\ 2S(\omega) & if \quad \omega < 0 \end{cases}$$

$$S(\omega) = \{S_R^+(\omega) + jS_I^+(\omega)\} + \{S_R^-(\omega) + jS_I^-(\omega)\} = S_f(\omega) + S_r(\omega)$$

DSP for Quadrature to Directional Signal Conversion

- **Time domain methods**
  - Phasing filter technique (PFT) (time domain Hilbert transform)
  - Weaver receiver technique
- **Frequency domain methods**
  - Frequency domain Hilbert transform
  - Complex FFT
  - Spectral translocation
- **Scale domain methods**
  - Complex continuous wavelet
  - Complex discrete wavelets

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Hilbert Transform...

- The Hilbert transform (HT) is a widely used frequency domain transform.
- It shifts the phase of positive frequency components by **-90^0** and negative frequency components by **$+90^0$**.
- The HT of a given function ***x*(*t*)** is defined by the convolution between this function and the impulse response of the HT (**$1/\pi t$**).

$$H[x(t)] = x(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau} d\tau$$

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...Hilbert Transform

- Specifically, if ***X*(*f*)** is the Fourier transform of ***x*(*t*)**, its Hilbert transform is represented by **$X_H(f)$**, where

$$X_H(f) = H[X(f)] = H_H(f)X(f) = (-j\,\mathrm{sgn}\, f)X(f)$$

- A **$\pm 90^0$** phase shift is equivalent to multiplying by **$e^{j90^0} = \pm j$**, so the transfer function of the HT **$H_H(f)$** can be written as

$$H_H(f) = -j\,\mathrm{sgn}\, f = \begin{cases} -j, f > 0 \\ +j, f < 0 \end{cases}$$

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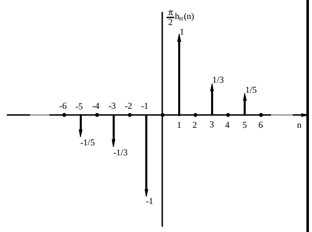
Impulse Response of HT

$$h_H(n) = \begin{cases} 0, n = 0 \\ \dfrac{2\sin^2(\pi n/2)}{\pi n}, n \neq 0 \end{cases}$$



An ideal HT filter can be approximated using standard filter design techniques.
If a FIR filter is to be used , only a finite number of samples of the impulse response suggested in the figure would be utilised.

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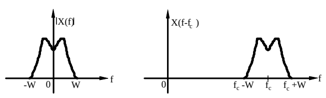
Complex Modulation

$$x(t)e^{j\omega_c t} \leftrightarrow X(\omega - \omega_c)$$



$$F\{\cos\omega_0 t\} = \pi\{\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\}$$

$$F\{\sin\omega_0 t\} = \frac{\pi}{j}\{\delta(\omega - \omega_0) - \delta(\omega + \omega_0)\}$$

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- **$x(t)e^{j\omega t}$** is not a real time function and cannot occur as a communication signal.
- However, signals of the form **$x(t)\cos(\omega t + \theta)$** are common and the related *modulation theorem* can be given as

$$x(t)\cos(\omega_c t + \theta) \leftrightarrow \frac{e^{j\theta}}{2} X(\omega - \omega_c) + \frac{e^{-j\theta}}{2} X(\omega + \omega_c)$$

- So, multiplying a band limited signal by a sinusoidal signal translates its spectrum up and down in frequency by **ω_c**

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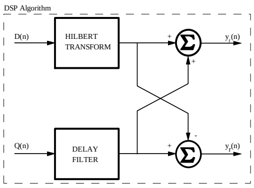
Asymmetrical implementation of the PFT

Doppler signal equation:

$$\begin{cases} D(n) = \pm s_f(n) \pm H[s_r(n)] \\ Q(n) = \pm H[s_f(n)] \pm s_r(n) \end{cases}$$



Mathematical description of algoritm:

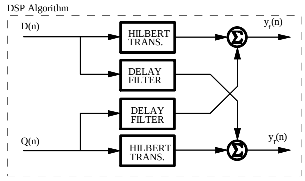
$$H[D(n)] = H[\pm s_f(n) \pm H[s_r(n)]] = \pm H[s_f(n)] \mp s_r(n)$$

$$\begin{cases} y_f(n) = Q(n) + H[D(n)] = \pm H[s_f(n)] \pm s_r(n) \pm H[s_f(n)] \mp s_r(n) \\ y_r(n) = Q(n) - H[D(n)] = \pm H[s_f(n)] \pm s_r(n) \mp H[s_f(n)] \pm s_r(n) \end{cases}$$

$$\begin{cases} y_f(n) = \pm 2H[s_f(n)] \\ y_r(n) = \pm 2 s_r(n) \end{cases}$$

Symmetrical implementation of the PFT

$$\begin{cases} D(n) = s_f(n) + H[s_r(n)] \\ Q(n) = H[s_f(n)] + s_r(n) \end{cases}$$

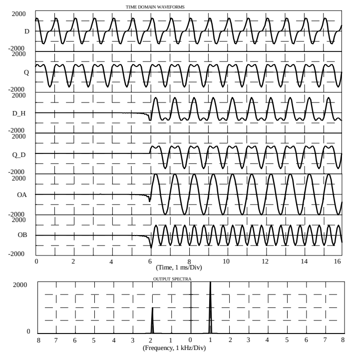


$$\boldsymbol{H[D(n)] = H[s_f(n) + H[s_r(n)]] = H[s_f(n)] - s_r(n)}$$

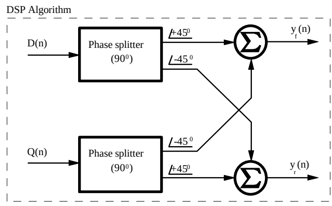
$$\boldsymbol{H[Q(n)] = H[H[s_f(n)] + s_r(n)] = -s_f(n) + H[s_r(n)]}$$

$$\begin{cases} y_f(n) = Q(n) + H[D(n)] = 2H[s_f(n)] \\ y_r(n) = D(n) + H[Q(n)] = 2H[s_r(n)] \end{cases} \qquad \begin{cases} y_f(n) = D(n) - H[Q(n)] = 2s_f(n) \\ y_r(n) = Q(n) - H[D(n)] = 2s_r(n) \end{cases}$$

Simulation result



- An alternative algorithm is to implement the HT using phase splitting networks
- A phase splitter is an all-pass filter which produces a quadrature signal pair from a single input
- The main advantage of this algorithm over the single filter HT is that the two filters have almost identical pass-band ripple characteristics



Weaver Receiver Technique (WRT)

- For a theoretical description of the system consider the quadrature Doppler signal defined by

$$\begin{cases} D(n) = s_f(n) + H[s_r(n)] \\ Q(n) = H[s_f(n)] + s_r(n) \end{cases}$$

which is band limited to $\boldsymbol{f_s/4}$, and a pair of quadrature pilot frequency signals given by

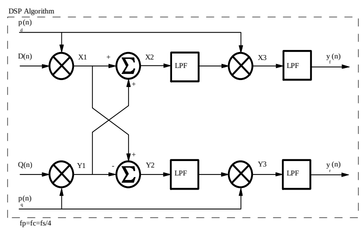
$$\boldsymbol{p_d(n) = \sin \omega_c n, \quad p_q(n) = \cos \omega_c n}$$

where $\boldsymbol{\omega_c/2\pi = f_s/4}$.

- The LPF is assumed to be an ideal LPF having a cut-off frequency of $\boldsymbol{f_s/4}$.

Asymmetrical implementation of the WRT

$$\begin{cases} D(n) = s_f(n) + H[s_r(n)] \\ Q(n) = H[s_f(n)] + s_r(n) \end{cases}$$



$$\boldsymbol{p_d(n) = \sin \omega_c n, \quad p_q(n) = \cos \omega_c n}$$

$$X2,Y2 = D(n).\,p_d(n) \pm Q(n).\,p_q(n)$$

$$= \{s_f(n).\sin\omega_c n + H[s_r(n)].\sin\omega_c n\} \pm \{H[s_f(n)].\cos\omega_c n + s_r(n)\cos\omega_c n\}$$

$$F\{s_f(n)\} = S_f(\omega) \ \ and \ \ F\{s_r(n)\} = S_r(\omega)$$

$$F\{H[s_f(n)]\} = H[S_f(\omega)] = \begin{cases} -jS_f(\omega), & 0 \le \omega < \pi \\ +jS_f(\omega), & -\pi \le \omega < 0 \end{cases}$$

$$\begin{cases} S_f^+(\omega) = S_f(\omega), 0 \le \omega < \pi \\ S_f^-(\omega) = S_f(\omega), -\pi \le \omega < 0 \end{cases}$$

$$F\{H[s_r(n)]\} = H[S_r(\omega)] = \begin{cases} -jS_r(\omega), & 0 \le \omega < \pi \\ +jS_r(\omega), & -\pi \le \omega < 0 \end{cases}$$

$$\begin{cases} S_r^+(\omega) = S_r(\omega), 0 \le \omega < \pi \\ S_r^-(\omega) = S_r(\omega), -\pi \le \omega < 0 \end{cases}$$

$$\begin{cases} S_f(\omega) = S_f^+(\omega) + S_f^-(\omega) \\ S_r(\omega) = S_r^+(\omega) + S_r^-(\omega) \end{cases} \qquad \begin{cases} H[S_f(\omega)] = -jS_f^+(\omega) + jS_f^-(\omega) \\ H[S_r(\omega)] = -jS_r^+(\omega) + jS_r^-(\omega) \end{cases}$$

$$F\{X2\} = \{-jS_f^+(\omega-\omega_c) + jS_f^-(\omega+\omega_c)\} + \{S_r^+(\omega+\omega_c) + S_r^-(\omega-\omega_c)\} \qquad = H[S_f(\omega_c-\omega)] + S_r(\omega_c+\omega)$$

$$F\{Y2\} = \{jS_f^+(\omega+\omega_c) - jS_f^-(\omega-\omega_c)\} + \{-S_r^+(\omega-\omega_c) - S_r^-(\omega+\omega_c)\} \qquad = -H[S_f(\omega_c+\omega)] - S_r(\omega_c-\omega)$$

$$F\{X2\} = H[S_f(\omega_c-\omega)] = -jS_f^+(\omega-\omega_c) + jS_f^-(\omega+\omega_c)$$

$$F\{Y2\} = -S_r(\omega_c-\omega) = -S_r^+(\omega-\omega_c) - S_r^-(\omega+\omega_c)$$

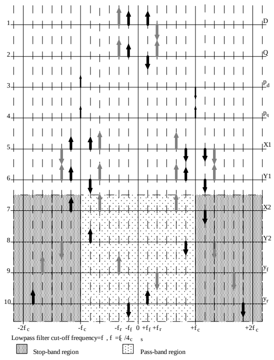
$$F\{X3\} = \frac{1}{2}S_f^+(\omega) + \frac{1}{2}S_f^-(\omega) - \frac{1}{2}S_f^+(\omega-2\omega_c) - \frac{1}{2}S_f^-(\omega+2\omega_c) \qquad = \frac{1}{2}S_f(\omega) - \frac{1}{2}S_f(2\omega_c-\omega)$$

$$F\{Y3\} = -\frac{1}{2}S_r^+(\omega) - \frac{1}{2}S_r^-(\omega) - \frac{1}{2}S_r^+(\omega-2\omega_c) - \frac{1}{2}S_r^-(\omega+2\omega_c) \qquad = -\frac{1}{2}S_r(\omega) - \frac{1}{2}S_r(2\omega_c-\omega)$$

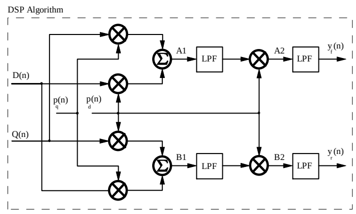
$$\begin{cases} F\{y_f(n)\} = \frac{1}{2}S_f(\omega), \\ F\{y_r(n)\} = -\frac{1}{2}S_r(\omega). \end{cases} \qquad \begin{cases} y_f(n) = F^{-1}\{\frac{1}{2}S_f(\omega)\} = \frac{1}{2}s_f(n), \\ y_r(n) = F^{-1}\{-\frac{1}{2}S_r(\omega)\} = -\frac{1}{2}s_r(n). \end{cases}$$

Graphical description

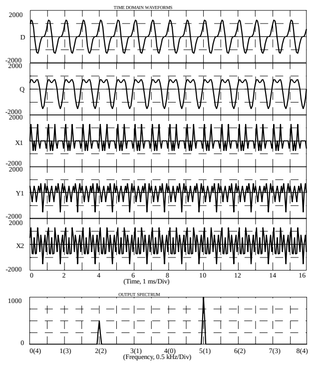
$\sin\omega_f t + \cos\omega_r t + j\cos\omega_f t + j\sin\omega_r t$



Symmetrical implementation



Simulation result



FREQUENCY DOMAIN PROCESSING

- These algorithms are almost entirely implemented in the frequency domain (after fast Fourier transform),
- They are based on the complex FFT process.
- The common steps for the all these implementations:
  - the complex FFT,
  - the inverse FFT
  - overlapping techniques to avoid Gibbs phenomena
- Three types of frequency domain algorithm will be described:
  Hilbert transform method,
  Complex FFT method, and
  Spectral translocation method.

FT of Complex Functions

- If $x(t)$ is a complex time function, i.e. $x(t)=x_r(t)+jx_i(t)$ where $x_r(t)$ and $x_i(t)$ are respectively the real part and imaginary part of the complex function $x(t)$,
- then the Fourier integral becomes

$$X(f)=\int_{-\infty}^{+\infty}[x_r(t)+jx_i(t)]e^{-j2\pi ft}dt=\int_{-\infty}^{+\infty}[x_r(t)\cos\omega t+x_i(t)\sin\omega t]dt$$

$$-j\int_{-\infty}^{+\infty}[x_r(t)\sin\omega t-x_i(t)\cos\omega t]dt=R(f)+jI(f)$$

$$F\{\cos\omega_0 t\}=\pi\{\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\}$$

$$F\{\sin\omega_0 t\}=\frac{\pi}{j}\{\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\}$$

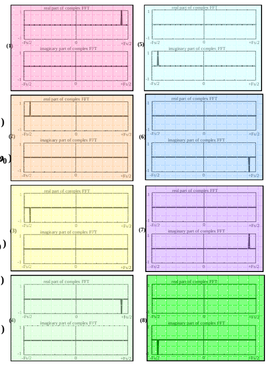
Properties of the Fourier transform for complex time functions

Time domain ($x(t)$)	**Frequency domain ($X(f)$)**
Real	Real part even, imaginary part odd
Imaginary	Real part odd, imaginary part even
Real even, imaginary odd	Real
Real odd, imaginary even	Imaginary
Real and even	Real and even
Real and odd	Imaginary and odd
Imaginary and even	Imaginary and even
Imaginary and odd	Real and odd
Complex and even	Complex and even
Complex and odd	Complex and odd

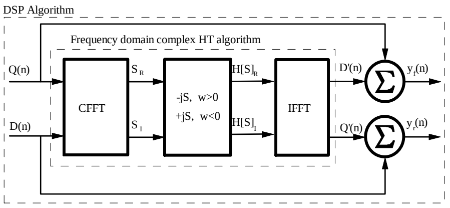
Interpretation of the complex Fourier transform

- If input of the complex Fourier transform is a complex quadrature time signal (specifically, a quadrature Doppler signal), it is possible to extract directional information by looking at its spectrum.
- Next, some results are obtained by calculating the complex Fourier transform for several combinations of the real and imaginary parts of the time signal (single frequency sine and cosine for simplicity).
- These results were confirmed by implementing simulations.

- **Case (1).**
  $x_r(t)=\cos\omega_0 t,\quad x_i(t)=\sin\omega_0 t,$
  $X(\omega)=F\{\cos\omega_0 t\}+jF\{\sin\omega_0 t\}=2\pi\delta(\omega-\omega_0)$
- **Case (2).**
  $x_r(t)=\cos\omega_0 t,\quad x_i(t)=-\sin\omega_0 t,$
  $X(\omega)=F\{\cos\omega_0 t\}+jF\{-\sin\omega_0 t\}=2\pi\delta(\omega+\omega_0)$
- **Case (3).**
  $x_r(t)=-\cos\omega_0 t,\quad x_i(t)=\sin\omega_0 t,$
  $X(\omega)=F\{-\cos\omega_0 t\}+jF\{\sin\omega_0 t\}=-2\pi\delta(\omega+\omega_0)$
- **Case (4).**
  $x_r(t)=-\cos\omega_0 t,\quad x_i(t)=-\sin\omega_0 t,$
  $X(\omega)=F\{-\cos\omega_0 t\}+jF\{-\sin\omega_0 t\}=-2\pi\delta(\omega-\omega_0)$
- **Case (5).**
  $x_r(t)=\sin\omega_0 t,\quad x_i(t)=\cos\omega_0 t,$
  $X(\omega)=F\{\sin\omega_0 t\}+jF\{\cos\omega_0 t\}=j2\pi\delta(\omega+\omega_0)$
- **Case (6).**
  $x_r(t)=\sin\omega_0 t,\quad x_i(t)=-\cos\omega_0 t,$
  $X(\omega)=F\{\sin\omega_0 t\}+jF\{-\cos\omega_0 t\}=-j2\pi\delta(\omega-\omega_0)$
- **Case (7).**
  $x_r(t)=-\sin\omega_0 t,\quad x_i(t)=\cos\omega_0 t,$
  $X(\omega)=F\{-\sin\omega_0 t\}+jF\{\cos\omega_0 t\}=j2\pi\delta(\omega-\omega_0)$
- **Case (8).**
  $x_r(t)=-\sin\omega_0 t,\quad x_i(t)=-\cos\omega_0 t,$
  $X(\omega)=F\{-\sin\omega_0 t\}+jF\{-\cos\omega_0 t\}=-j2\pi\delta(\omega+\omega_0)$

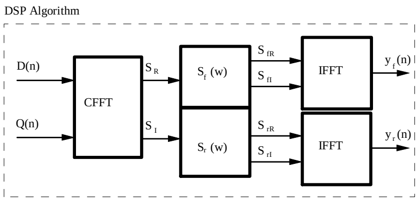


Frequency domain Hilbert transform algorithm



Complex FFT Method (CFFT)

- The complex FFT has been used to separate the directional signal information from quadrature signals
  - so that the spectra of the directional signals can be estimated and displayed as sonograms.
- It can be shown that the phase information of the directional signals is well preserved and can be used to recover these signals.



$$s(n) \equiv D(n) + jQ(n) \equiv \{s_f(n) + H[s_r(n)]\} + j\{H[s_f(n)] + s_r(n)\}$$
$$\equiv \{s_f(n) + jH[s_f(n)]\} + j\{s_r(n) - jH[s_r(n)]\}$$

$$F\{[s(n)\} = \begin{cases} \{S_f(\omega) + S_f(\omega)\} + j\{S_r(\omega) - S_r(\omega)\}, 0 \le \omega < \pi \\ \{S_f(\omega) - S_f(\omega)\} + j\{S_r(\omega) + S_r(\omega)\}, -\pi \le \omega < 0 \end{cases}$$

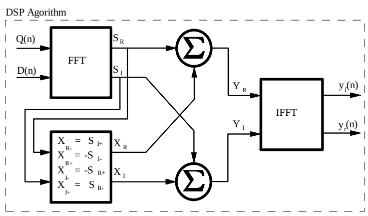
$$F\{s(n)\} = S(\omega) = \begin{cases} 2S_f(\omega), 0 \le \omega < \pi \\ j2S_r(\omega), -\pi \le \omega < 0 \end{cases}$$

$$S^+(\omega) = \begin{cases} S(\omega), 0 \le \omega < \pi \\ 0, -\pi \le \omega < 0 \end{cases} \qquad S^-(\omega) = \begin{cases} 0, 0 \le \omega < \pi \\ S(\omega), -\pi \le \omega < 0 \end{cases}$$

$$\Re\{S_f(\omega)\} = \begin{cases} \Re\{S^+(\omega)\}, 0 \le \omega < \pi \\ \Re\{S^+(-\omega)\}, -\pi \le \omega < 0 \end{cases} \qquad \Im\{S_f(\omega)\} = \begin{cases} \Im\{S^+(\omega)\}, 0 \le \omega < \pi \\ -\Im\{S^+(-\omega)\}, -\pi \le \omega < 0 \end{cases}$$

$$\Re\{S_r(\omega)\} = \begin{cases} \Im\{S^-(-\omega)\}, 0 \le \omega < \pi \\ \Im\{S^-(\omega)\}, -\pi \le \omega < 0 \end{cases} \qquad \Im\{S_r(\omega)\} = \begin{cases} \Re\{S^-(-\omega)\}, 0 \le \omega < \pi \\ -\Re\{S^-(\omega)\}, -\pi \le \omega < 0 \end{cases}$$

Spectral Translocation Method (STM)



$$F\{s(n)\} = S(\omega) = \begin{cases} 2S_f(\omega), 0 \le \omega < \pi \\ j2S_r(\omega), -\pi \le \omega < 0 \end{cases}$$

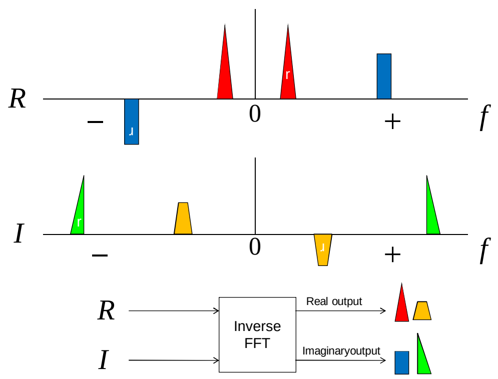
$$S(\omega) = 2S_f^+(\omega) + j2S_r^-(\omega) = \Re\{S^+(\omega) + S^-(\omega)\} + j\Im\{S^+(\omega) + S^-(\omega)\}$$

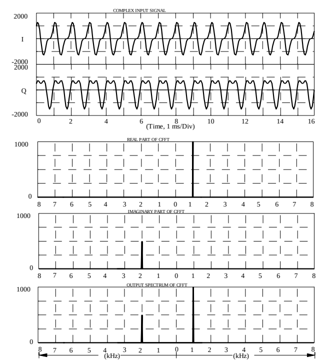
$$S^-(-\omega) = S^+(\omega), \quad S^+(-\omega) = S^-(\omega)$$

$$X(\omega) \equiv \Re\{S^+(-\omega) - S^-(-\omega)\} + j\Im\{-S^+(-\omega) + S^-(-\omega)\}$$
$$\equiv \Re\{-S^+(\omega) + S^-(\omega)\} + j\Im\{S^+(\omega) - S^-(\omega)\}$$

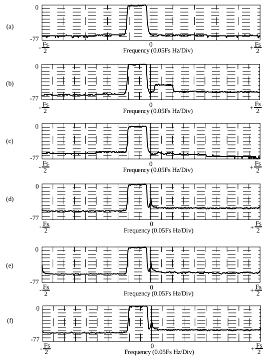
$$\begin{aligned} Y(\omega) &= S(\omega) + X(\omega) \\ &\equiv \{\Re\{S(\omega)\} + \Re\{X(\omega)\}\} + j\{\Im\{S(\omega)\} + \Im\{X(\omega)\}\} \\ &= 2S_f^+(\omega) + j2S_r^-(\omega) + 2S_f^-(\omega) + j2S_r^+(\omega) \\ &= 2\{S_f^+(\omega) + S_f^-(\omega)\} + j2\{S_r^+(\omega) + S_r^-(\omega)\} \\ &= 2S_f(\omega) + j2S_r(\omega) \end{aligned}$$

$$y(n) = 2s_f(n) + j2s_r(n)$$





- The averaged output power spectra of the simulations for;
  - (a) the reference quadrature signal,
  - (b) the PFT,
  - (c) the EWRT,
  - (d) the HTM,
  - (e) the CFFT and
  - (f) the STM.
- Magnitude scale is 7.7 dB/Div.



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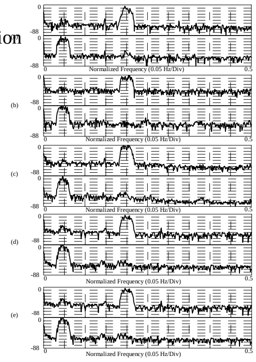
- Results for time domain separation techniques. Logarithmic scale output spectra of practical implementations for;
  - (a) PFT;
  - (b) EWRT;
  - (c) HTM;
  - (d) complex FFT;
  - (e) STM
- when the input is a narrow band quadrature signal containing the forward and reverse flow components (magnitude scale is 8.8 dB/Div).



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