Advanced Digital Signal Processing

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Digital Signal Processing (DSP)

Basics:
What is DSP?

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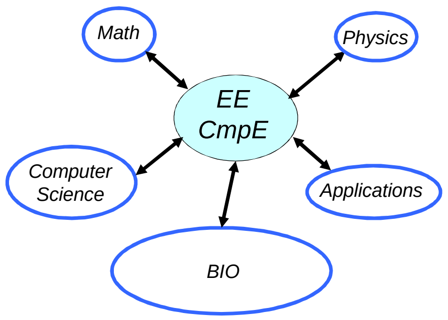
CONVERGING FIELDS



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Digital Signal Processing (DSP)

Dictionary definitions of the words in DSP:

- *Digital*
  - *operating by the use of discrete signals to represent data in the form of numbers*
- *Signal*
  - *a variable parameter by which information is conveyed through an electronic circuit*
- *Processing*
  - *to perform operations on data according to programmed instructions*
- **So a simple definition of DSP could be:**
  - **changing or analysing information which is measured as discrete sequences of numbers**
- **Unique features of DSP as opposed to ordinary digital processing:**
  - **signals come from the real world**
    - **this intimate connection with the real world leads to many unique needs such as the need to react in real time and a need to measure signals and convert them to digital numbers**
  - **signals are discrete**
    - **which means the information in between discrete samples is lost**

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WHY USE DSP ?

- **Versatility:**
  - digital systems can be reprogrammed for other applications
  - digital systems can be ported to different hardware
- **Repeatability:**
  - digital systems can be easily duplicated
  - digital systems do not depend on strict component tolerances
  - digital system responses do not drift with temperature
- **Simplicity:**
  - some things can be done more easily digitally than with analogue systems

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DSP is used in a very wide variety of applications

- Radar, sonar, telephony, audio, multimedia, communications, ultrasound, process control, digital camera, digital tv, Telecommunications, Sound & Music, Fourier Optics, X-ray Crystallography, Protein Structure & DNA, Computerized Tomography, Nuclear Magnetic Resonance: MRI,Radioastronomy
- All these applications share some common features:
  - they use a lot of maths (multiplying and adding signals)
  - they deal with signals that come from the real world
  - they require a response in a certain time
- Where general purpose DSP processors are concerned, most applications deal with signal frequencies that are in the audio range

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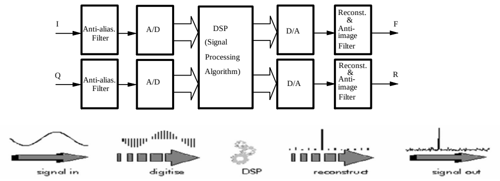
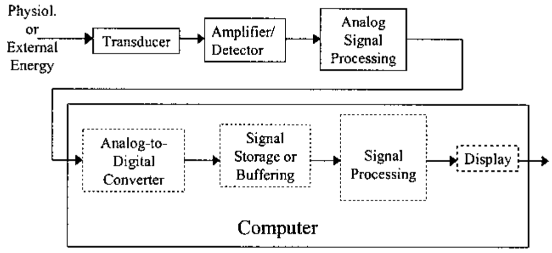
Fundamental concepts in DSP

- DSP applications deal with analogue signals
  - the analogue signal has to be converted to digital form



A typical biomedical measurement system



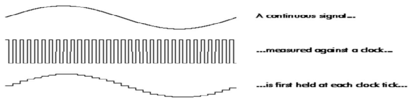
Transducers

- A "transducer" is a device that converts energy from one form to another.
- In signal processing applications, the purpose of energy conversion is to transfer information, not to transform energy.
- In physiological measurement systems, transducers may be
  - input transducers (or sensors)
    - they convert a non-electrical energy into an electrical signal.
    - for example, a microphone.
  - output transducers (or actuators)
    - they convert an electrical signal into a non-electrical energy.
    - For example, a speaker.

- The analogue signal
  - a continuous variable defined with infinite precision

  is converted to a discrete sequence of measured values which are represented digitally
- Information is lost in converting from analogue to digital, due to:
  - inaccuracies in the measurement
  - uncertainty in timing
  - limits on the duration of the measurement
- These effects are called quantisation errors

- The continuous analogue signal has to be held before it can be sampled

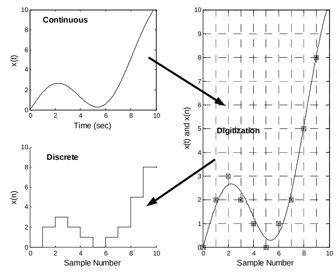


- Otherwise, the signal would be changing during the measurement
- Only after it has been held can the signal be measured, and the measurement converted to a digital value

Signal Encoding: Analog-to Digital Conversion

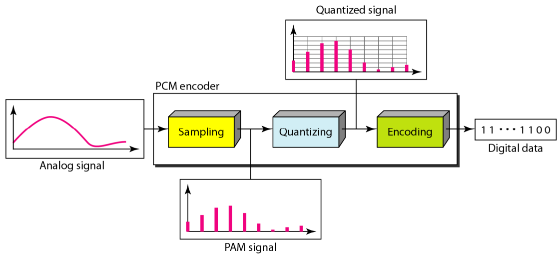
Continuous (analog) signal ↔ Discrete signal

$x(t) = f(t)$ ↔ Analog to digital conversion ↔ $x(n) = x(1), x(2), x(3), \ldots x(n)$



Analog-to Digital Conversion

- ADC consists of four steps to digitize an analog signal:
  1. Filtering
  2. Sampling
  3. Quantization
  4. Binary encoding
- Before we sample, we have to filter the signal to limit the maximum frequency of the signal as it affects the sampling rate.
- Filtering should ensure that we do not distort the signal, ie remove high frequency components that affect the signal shape.

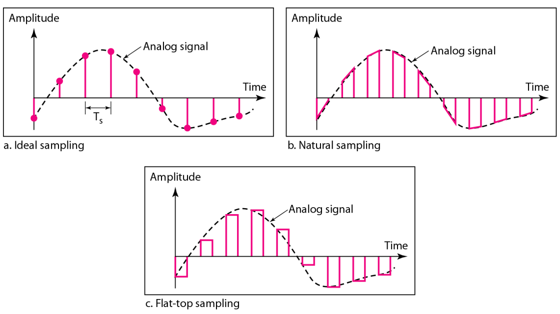


Sampling

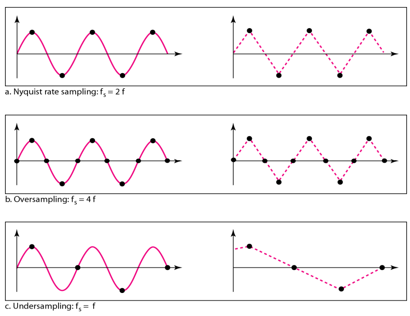
- The sampling results in a discrete set of digital numbers that represent measurements of the signal
  - usually taken at equal intervals of time
- Sampling takes place after the hold
  - The hold circuit must be fast enough that the signal is not changing during the time the circuit is acquiring the signal value
- We don't know what we don't measure
- In the process of measuring the signal, some information is lost

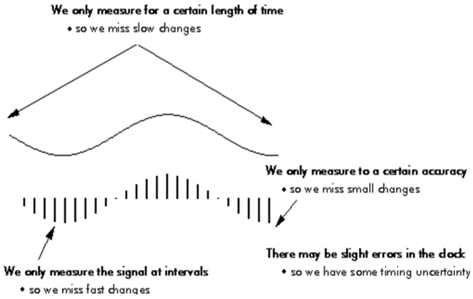
Sampling

- Analog signal is sampled every T_S secs.
- T_s is referred to as the sampling interval.
- $f_s = 1/T_s$ is called the sampling rate or sampling frequency.
- There are 3 sampling methods:
  - Ideal - an impulse at each sampling instant
  - Natural - a pulse of short width with varying amplitude
  - Flattop - sample and hold, like natural but with single amplitude value
- The process is referred to as pulse amplitude modulation PAM and the outcome is a signal with analog (non integer) values



Recovery of a sampled sine wave for different sampling rates





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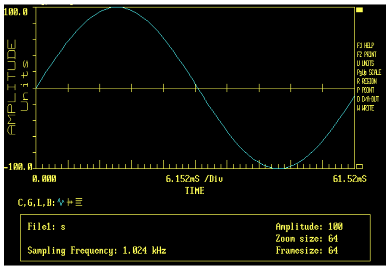

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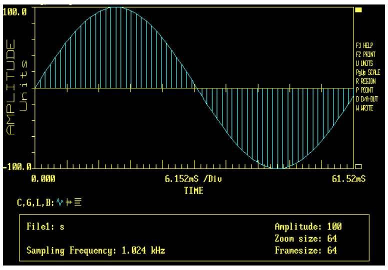

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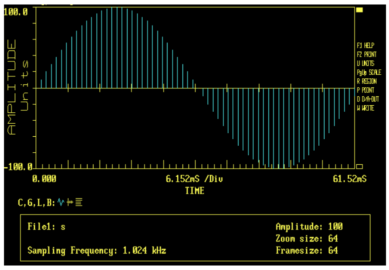

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Sampling Theorem

$$F_s \geq 2f_m$$

According to the Nyquist theorem, the sampling rate must be at least 2 times the highest frequency contained in the signal.

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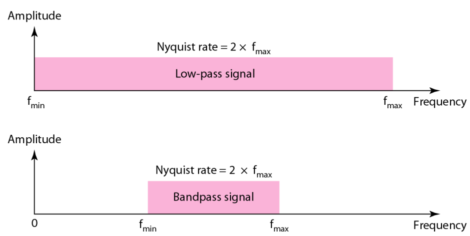
Nyquist sampling rate for low-pass and bandpass signals



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Quantization

- Sampling results in a series of pulses of varying amplitude values ranging between two limits: a min and a max.
- The amplitude values are infinite between the two limits.
- We need to map the *infinite* amplitude values onto a finite set of known values.
- This is achieved by dividing the distance between min and max into L zones, each of height Δ.

$$\Delta = (max - min)/L$$

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Quantization Levels

- The midpoint of each zone is assigned a value from 0 to L-1 (resulting in L values)
- Each sample falling in a zone is then approximated to the value of the midpoint.

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Quantization Zones

- Assume we have a voltage signal with amplitudes V_{min}=-20V and V_{max}=+20V.
- We want to use L=8 quantization levels.
- Zone width Δ = (20 - -20)/8 = 5
- The 8 zones are: -20 to -15, -15 to -10, -10 to -5, -5 to 0, 0 to +5, +5 to +10, +10 to +15, +15 to +20
- The midpoints are: -17.5, -12.5, -7.5, -2.5, 2.5, 7.5, 12.5, 17.5

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Assigning Codes to Zones

- Each zone is then assigned a binary code.
- The number of bits required to encode the zones, or the number of bits per sample as it is commonly referred to, is obtained as follows:

$$n_b = \log_2 L$$

- Given our example, $n_b = 3$
- The 8 zone (or level) codes are therefore: 000, 001, 010, 011, 100, 101, 110, and 111
- Assigning codes to zones:
  - 000 will refer to zone -20 to -15
  - 001 to zone -15 to -10, etc.

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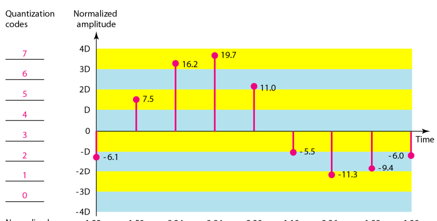
Quantization and encoding of a sampled signal



Normalized PAM values	-1.22	1.50	3.24	3.94	2.20	-1.10	-2.26	-1.88	-1.20
Normalized quantized values	-1.50	1.50	3.50	3.50	2.50	-1.50	-2.50	-1.50	-1.50
Normalized error	-0.38	0	+0.26	-0.44	+0.30	-0.40	-0.24	+0.38	-0.30
Quantization code	2	5	7	7	6	2	1	2	2
Encoded words	010	101	111	111	110	010	001	010	010

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Quantization Error

- When a signal is quantized, we introduce an error - the coded signal is an approximation of the actual amplitude value.
- The difference between actual and coded value (midpoint) is referred to as the quantization error.
- The more zones, the smaller Δ which results in smaller errors.
- BUT, the more zones the more bits required to encode the samples -> higher bit rate

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Analog-to-digital Conversion

Example An 12-bit analog-to-digital converter (ADC) advertises an accuracy of ± the least significant bit (LSB). If the input range of the ADC is 0 to 10 volts, what is the accuracy of the ADC in analog volts?

Solution: If the input range is 10 volts then the analog voltage represented by the LSB would be:

$$V_{LSB} = \frac{V_{max}}{2^{\text{Nu bits}}} = \frac{10}{2^{12}} = \frac{10}{4096} = .0024 \text{ volts}$$

Hence the accuracy would be ± .0024 volts.

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Sampling related concepts

- Over/exact/under sampling
- Regular/irregular sampling
- Linear/Logarithmic sampling
- Aliasing
- Anti-aliasing filter
- Image
- Anti-image filter

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Steps for digitization/reconstruction of a signal

- Band limiting (LPF)
- Sampling / Holding
- Quantization
- Coding

These are basic steps for A/D conversion

- D/A converter
- Sampling / Holding
- Image rejection

These are basic steps for reconstructing a sampled digital signal

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Digital data: end product of A/D conversion and related concepts

- Bit: least digital information, binary 1 or 0
- Nibble: 4 bits
- Byte: 8 bits, 2 nibbles
- Word: 16 bits, 2 bytes, 4 nibbles
- Some jargon:
  - integer, signed integer, long integer, 2s complement, hexadecimal, octal, floating point, etc.

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Data types

- Our first requirement is to find a way to represent information (data) in a form that is mutually comprehensible by human and machine.
  - Ultimately, we will have to develop schemes for representing all conceivable types of information - language, images, actions, etc.
  - We will start by examining different ways of representing *integers*, and look for a form that suits the computer.
  - Specifically, the devices that make up a computer are switches that can be on or off, i.e. at high or low voltage. Thus they naturally provide us with two symbols to work with: we can call them *on* & *off*, or (more usefully) *0* and *1*.

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Signal

- An information variable represented by physical quantity.
- For digital systems, the variable takes on discrete values.
- Two level, or binary values are the most prevalent values in digital systems.
- Binary values are represented abstractly by:
  - digits 0 and 1
  - words (symbols) False (F) and True (T)
  - words (symbols) Low (L) and High (H)
  - and words On and Off.
- Binary values are represented by values or ranges of values of physical quantities

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Number Systems – Representation

- Positive radix, positional number systems
- A number with *radix* **r** is represented by a string of digits:

$$A_{n-1}A_{n-2}\ldots A_1A_0 \cdot A_{-1}A_{-2}\ldots A_{-m+1}A_{-m}$$

in which $0 \le A_i < r$ and $\cdot$ is the *radix point*.

- The string of digits represents the power series:

$$(\text{Number})_r = \left(\sum_{i=0}^{i=n-1} A_i \cdot r^i\right) + \left(\sum_{j=-m}^{j=-1} A_j \cdot r^j\right)$$

(Integer Portion) + (Fraction Portion)

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Decimal Numbers

- "decimal" means that we have ten digits to use in our representation (the symbols 0 through 9)
- What is 3546?
  - it is *three* thousands plus *five* hundreds plus *four* tens plus *six* ones.
  - i.e. $3546 = 3.10^3 + 5.10^2 + 4.10^1 + 6.10^0$
- How about negative numbers?
  - we use two more symbols to distinguish positive and negative:

  \+ and -

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Unsigned Binary Integers

$Y = \text{“abc”} = a.2^2 + b.2^1 + c.2^0$

(where the digits a, b, c can each take on the values of 0 or 1 only)

N = number of bits

Range is:
$0 \le i < 2^N - 1$

	3-bits	5-bits	8-bits
0	000	00000	00000000
1	001	00001	00000001
2	010	00010	00000010
3	011	00011	00000011
4	100	00100	00000100

Problem:
- How do we represent *negative* numbers?

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Two's Complement

- Transformation
  - To transform a into -a, invert all bits in a and add 1 to the result

Range is:
$-2^{N-1} < i < 2^{N-1} - 1$

Advantages:
- Operations need not check the sign
- Only one representation for zero
- Efficient use of all the bits

-16	10000
…	…
-3	11101
-2	11110
-1	11111
0	00000
+1	00001
+2	00010
+3	00011
…	…
+15	01111

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Limitations of integer representations

- Most numbers are not integer!
  - Even with integers, there are two other considerations:
- Range:
  - The magnitude of the numbers we can represent is determined by how many bits we use:
    - e.g. with 32 bits the largest number we can represent is about +/- 2 billion, far too small for many purposes.
- Precision:
  - The exactness with which we can specify a number:
    - e.g. a 32 bit number gives us 31 bits of precision, or roughly 9 figure precision in decimal repesentation.
- We need another data type!

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Real numbers

- Our decimal system handles non-integer *real* numbers by adding yet another symbol - the decimal point (.) to make a *fixed point* notation:
  - e.g. $3456.78 = 3.10^3 + 4.10^2 + 5.10^1 + 6.10^0 + 7.10^{-1} + 8.10^{-2}$
- The *floating point*, or scientific, notation allows us to represent very large and very small numbers (integer or real), with as much or as little precision as needed:
  - Unit of electric charge e = 1.602 176 462 x 10^{-19} Coulomb
  - Volume of universe = 1 x 10^{85} cm^3
    - the two components of these numbers are called the mantissa and the exponent

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Real numbers in binary

- We mimic the decimal floating point notation to create a "hybrid" binary floating point number:
  - We first use a "binary point" to separate whole numbers from fractional numbers to make a fixed point notation:
    - e.g. $00011001.110 = 1.2^4 + 1.2^3 + 1.2^1 + 1.2^{-1} + 1.2^{-2}$ => 25.75
      ($2^{-1} = 0.5$ and $2^{-2} = 0.25$, etc.)
  - We then "float" the binary point:
    - 00011001.110 => 1.1001110 x 2^4
      mantissa = 1.1001110, exponent = 4
  - Now we have to express this without the extra symbols (x, 2, .)
    - by convention, we divide the available bits into three fields:

sign, mantissa, exponent

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IEEE-754 fp numbers - 1

s	biased exp.	fraction
1	8 bits	23 bits

32 bits:

$$N = (-1)^s \times 1.\text{fraction} \times 2^{(\text{biased exp.} - 127)}$$

- Sign: 1 bit
- Mantissa: 23 bits
  - We "normalize" the mantissa by dropping the leading 1 and recording only its fractional part (why?)
- Exponent: 8 bits
  - In order to handle both +ve and -ve exponents, we add 127 to the actual exponent to create a "biased exponent":
    - 2^{-127} => biased exponent = 0000 0000 (= 0)
    - 2^0 => biased exponent = 0111 1111 (= 127)
    - 2^{+127} => biased exponent = 1111 1110 (= 254)

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IEEE-754 fp numbers - 2

- Example: Find the corresponding fp representation of 25.75
  - 25.75 => 00011001.110 => 1.1001110 x 2^4
  - sign bit = 0 (+ve)
  - normalized mantissa (fraction) = 100 1110 0000 0000 0000 0000
  - biased exponent = 4 + 127 = 131 => 1000 0011
  - so 25.75 => 0 1000 0011 100 1110 0000 0000 0000 0000 => x41CE0000
- Values represented by convention:
  - Infinity (+ and -): exponent = 255 (1111 1111) and fraction = 0
  - NaN (not a number): exponent = 255 and fraction ≠ 0
  - Zero (0): exponent = 0 and fraction = 0
    - note: exponent = 0 => fraction is *de-normalized*, i.e no hidden 1

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Binary Numbers and Binary Coding

- **Flexibility of representation**
  - **Within constraints below, can assign any binary combination (called a code word) to any data as long as data is uniquely encoded.**
- **Information Types**
  - **Numeric**
    - **Must represent range of data needed**
    - **Very desirable to represent data such that simple, straightforward computation for common arithmetic operations permitted**
    - **Tight relation to binary numbers**
  - **Non-numeric**
    - **Greater flexibility since arithmetic operations not applied.**
    - **Not tied to binary numbers**

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Non-numeric Binary Codes

- **Given *n* binary digits (called bits), a binary code is a mapping from a set of represented elements to a subset of the 2^n binary numbers.**
- **Example: A binary code for the seven colors of the rainbow**
- **Code 100 is not used**

Color	Binary Number
Red	000
Orange	001
Yellow	010
Green	011
Blue	101
Indigo	110
Violet	111

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Number of Bits Required

- Given M elements to be represented by a binary code, the minimum number of bits, *n*, needed, satisfies the following relationships:

  $2^n > M > 2^{(n-1)}$

  $n = \lceil \log_2 M \rceil$ where $\lceil x \rceil$, called the *ceiling function,* is the integer greater than or equal to *x*.

- Example: How many bits are required to represent decimal digits with a binary code?
  - 4 bits are required ($n = \lceil \log_2 9 \rceil = 4$)

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